Summary of attached articles

We here briefly summarized (in English) the papers which have been referenced in this synthesis.

Semi–parametric estimation

We have considered three semi–parametric estimation problems :

1 Estimation of the frequency of an irregularly sampled periodic function, [8].

2 Wavelet methods for estimating the long memory parameter of Gaussian and linear processes [13, 11, 12] ;

3 Estimation of the tail index of an Infinite source Poisson process [6].

In [8] the model

$$Y_j = s_*(X_j) + \epsilon_j , \quad j = 1 \ldots , n ,$$

is considered, where s_* is an unknown periodic function, X_j is a renewal process (having i.i.d. positive increments) and $\{\epsilon_j , j \geq 1\}$ is a Gaussian white noise, independent of $\{X_j , j \geq 1\}$. We consider the problem of estimating the frequency of s_* when $(X_j , Y_j), j = 1, \ldots n$ is observed. We use the Lomb-Scargle periodogram, exhibit a consistent estimator and show a central limit theorem for this estimator.

In [13] the second order properties of the discrete wavelet transform of a long memory process are studied and used to evaluate the mean square error of a wavelet log-regression estimator of the long memory parameter of a Gaussian process. A central limit theorem for this estimator is obtained in [11]. Another wavelet estimator is considered in [12], which corresponds to a Whittle contrast approach in the wavelet domain. In this paper the rate of convergence is derived for a wide class of long memory linear processes and a central limit theorem is obtained in the Gaussian case.

Finally in [6], we estimate the tail index of the durations of the pulses of a shot–noise

$$X(t) = \sum_k U_k \mathbb{1}(T_k \leq t < T_k + \eta_k) ,$$

where $\{T_k\}$ are Poisson arrival times and $\{(U_k , \eta_k)\}$ are i.i.d.marks, independent of the arrival times satisfying

$$\mathbb{E}[U_0^p \mathbb{1}_{\eta_0 > t}] = L_p(t)\ t^{-\alpha} ,$$

with $p = 0, 1, \ldots , 4$, L_p slowly varying as $t \to \infty$ and $\alpha \in (0, 2)$. The estimator only depends on (possibly discretely sampled) observations of $\{X_t , t = 0, \ldots , T\}$. It is adapted from [12] as it relies on the long memory behavior of this process. The consistency and the rate of covariance are investigated.

Non–parametric estimation

We have considered three non–parametric estimation problems :

1 Estimation of the density of the pulse energy distribution of a shot noise process [15, 10].

2 Estimation of the density of a mixing distribution for discrete mixtures [14].

3 Estimation of a tvAR process [9] ;

In [15, 10], we consider a shot-noise model with Poisson arrivals

$$S(t) = \sum_k Z_k(t - T_k) ,$$

where Z_k are i.i.d. and independent of the Poisson arrivals $\{T_k\}$. We wish to estimate the distribution of the variables

$$Y_k = \int_0^\infty Z_k(u)\,\mathrm{d}u$$

from observations of $\{S(t), t \in (0,T)\}$. We study two estimators based on the cycles of the process S (On and Off periods).

In [14], we study a projection estimator of the density f of a mixing distribution with respect to a given dominating measure ν, obtained from n i.i.d. random variables $X_1, \ldots, X_n$ having the mixture distribution

$$\pi_f = \int \pi_\theta\ f(\theta)\ \nu(\mathrm{d}\theta)\ ,$$

where $\{\pi_\theta\}$ is a known family of mixand distributions. We focus on the case where this family is a discrete distribution and obtain minimax bounds for estimating f.

In [9], we consider a time varying auto-regressive (TVAR) model defined by

$$X_{k+1,n} = \boldsymbol{\theta}_{k,n}^t \mathbf{X}_{k,n} + \sigma_{k+1,n}\epsilon_{k+1,n}\ ,$$

where

(i) we denoted $\mathbf{X}_{k,n} = [X_{k,n}\ \ \ldots\ \ X_{k-p,n}]^T$,

(ii) the array $\{\epsilon_{k,n},\ k \in \mathbb{N}, n \geq 1\}$ is a centered white noise with unit variance,

(iii) the time varying coefficients are given by $\boldsymbol{\theta}_{k,n} = \boldsymbol{\theta}(k/n)$ with $\boldsymbol{\theta} : \mathbb{R}_+ \to \mathbb{R}^p$ and $\sigma_{k,n} = \sigma(k/n)$ with $\sigma : \mathbb{R}_+ \to \mathbb{R}_+$.

We give some conditions for the stability of this model as $n \to \infty$ and we study a recursive estimator from the observations $X_{k,n}$, $k = 1, \ldots, n$ defined by

$$\hat{\boldsymbol{\theta}}_{k+1,n} = \hat{\boldsymbol{\theta}}_{k,n} + \mu \frac{(X_{k+1,n} - \hat{\boldsymbol{\theta}}_{k,n}^T \mathbf{X}_{k,n})\mathbf{X}_{k,n}}{1 + \mu|\mathbf{X}_{k,n}|^2}\ , \tag{1}$$

where $\mu > 0$ is the step-size tuned by the user and $|\cdot|$ denotes the Euclidean norm.

Probabilistic models and regularity issues

1 Hausdorff dimension of random wavelet series [1].

2 Linear fractional stable sheet [3, 2].

3 Scaling dead leaves model [7, 4].

In [1], we are interested in the Hausdorff dimension of the graph of a random process. A new formulation of the Frostman criterion is proposed for bounding this dimension from below. It is then shown that this formulation allows to improve some previous results obtained when the underlying process is a random wavelet series admitting some form of scaling properties at small scales.

In [3, 2], we study the modulus of continuity and the existence of the local time for linear fractional stable sheet (LFSS). The estimates on the modulus of continuity are obtained by using a random wavelet series representation of the LFSS.

In [4], we provide some new results on the dead leaves model, a model obtain by randomly superimposing random object in the space, resulting in a random tessellation. For instance we completely characterize the distribution of its boundary. In [7], these results are used to investigate some *scaling* dead leaves models obtained by letting converge a dead leaves model as the objects sizes distribution converges to a power law, allowing a scaling behavior in the limit, either at small scales or at large scales. We then study the regularity properties of the limit in the most interesting case, having in mind the problem of modeling natural images.

Long range dependence

1 Gaussian and linear models [13, 11, 12] ;

2 Long memory of the infinite source Poisson process [6].

3 Existence of some ARCH(∞) and IARCH(∞) processes, [5].

We already mentioned [13, 11, 12, 6] as they are mainly concerned with semi–parametric estimation.

In [5], we provide new sufficient conditions for the existence of some ARCH(∞) process. These conditions include the case of the FIGARCH$(0, d, 0)$ process, $d \in (0, 1)$, a process whose existence was not rigorously proven to our knowledge. However its long range memory properties are still be established.

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