

Introduction to Magnetic Resonance Imaging (MRI)

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Summary

- 1 Introduction
- 2 Nuclear Magnetic Resonance (NMR)
- 3 Net magnetization
- 4 Relaxation
- 5 Free induction decay (FID)
- 6 Image contrast
- 7 Spatial localization
- 8 Recap
- 9 MRI artifacts
- 10 MRI modalities

Magnetic Resonance Imaging

It is a medical imaging technique and diagnostic tool in radiology based on the Nuclear Magnetic Resonance (NMR).

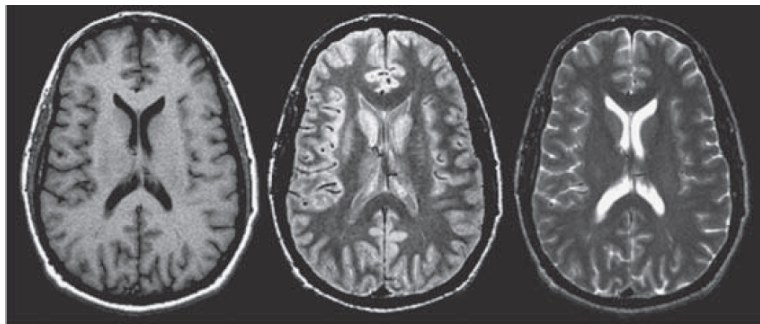
Main pros

- non-invasive and 3D
- no ionizing radiations
- very good spatial resolution (1mm isotropic for anatomy)
- anatomy, functional and physiological state of the internal organs
- both healthy and pathological tissues

Main cons

- expensive
- long scan time (20-45 minutes)
- no metal (pacemaker, valves, etc.)
- not comfortable for the patient and loud

Magnetic Resonance Imaging



T_1 -weighted
(TR = 600, TE = 11)

Density-weighted
(TR = 3000, TE = 17)

T_2 -weighted
(TR = 3800, TE = 102)

Figure 1: Magnetic resonance images taken from the same anatomical section of the human brain. Contrast between different tissues can change simply by varying some parameters of the acquisition such as the Repetition Time (TR) and the Echo Time (TE). Image taken from [1].



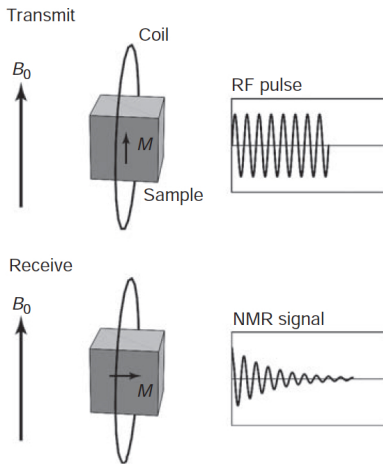
It is composed of:

- **a magnet** which produces a very powerful uniform magnetic field B_0 (1.5T - 11T, note that the earth magnetic field is around 60 microT)
- **shim coils** to make B_0 homogeneous
- **gradient coils** that make the magnetic field linearly vary across the imaging volume. This determines the plane of imaging
- **Radio Frequency (RF) transmission system**

Summary

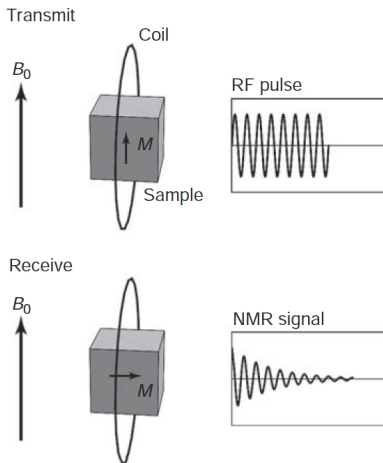
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Nuclear Magnetic Resonance (NMR)



- A sample is placed in a large uniform magnetic field B_0
- An oscillating current is applied to the coil for few milliseconds, which produces an oscillating magnetic field B_1 in the sample (Jefimenko's equations).
- The oscillations of the magnetic field are in the radiofrequency (RF) range (3 kHz to 300 GHz)

Nuclear Magnetic Resonance (NMR)



- Certain atomic nuclei, such as the hydrogen, are able to absorb specific radio frequency energy and emit a small portion of that energy.
- The produced time-varying magnetic field will induce voltage in the coil (Maxwell–Faraday equation)
- This current, oscillating at the same frequency as the RF pulse, is the NMR signal

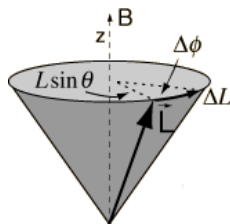
Recap about Physics

- The **atomic nucleus** is the small, dense region consisting of protons and neutrons at the center of an atom.
- **Spin** is an intrinsic form of angular momentum carried by the atomic nuclei. Its magnitude cannot be changed, only its direction can vary. Only nuclei with an odd number of protons and neutrons have a net spin, such as hydrogen (only one proton).
- Nuclei with spin can be seen as tiny magnets, whose south-north axis is parallel to the spin axis.
- They possess a **magnetic dipole moment**, namely a vector μ which points from the south to north pole of the magnet. It is described by:

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}_0 = \gamma \mathbf{L} \times \mathbf{B}_0 \quad (1)$$

where $\boldsymbol{\tau}$ is the torque (couple) acting on the magnet, γ is the gyromagnetic ratio and \mathbf{L} is the angular momentum.

Recap about Physics



Remember that $\boldsymbol{\tau} = \frac{d\mathbf{L}}{dt}$

$$\boldsymbol{\tau} = \frac{d\mathbf{L}}{dt} = \frac{L \sin(\theta) d\phi}{dt} = \gamma L B_0 \sin(\theta) \quad (2)$$

Thus

$$\frac{d\phi}{dt} = \omega_L = \gamma B_0 \rightarrow f_L = \frac{\gamma B_0}{2\pi} \quad (3)$$

This is the precession angular frequency, namely the **Larmor frequency**.

Nuclear Magnetic Resonance (NMR)

This means that when a nucleus with spin is placed into a magnetic field \mathbf{B}_0 , the magnetic field will exert a torque on the nucleus making it precess (rotate) around the field.

Resonant frequency of NMR

The Larmor frequency f_L is the resonant frequency of NMR and it is directly proportional to the magnitude of \mathbf{B}_0 ($f_L = \frac{\gamma B_0}{2\pi}$). Magnetic resonance absorption will only occur at this frequency !

Nuclear Magnetic Resonance (NMR)

Nucleus	γ (MHz/T)
1H	42.58
^{13}C	10.71

- This means that for a $B_0 = 1.5T$, if we put a 1H nucleus inside, we will obtain $\omega_L = 63,86MHz$.
- Why 1H ?

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- Because the human body is composed of 50-65% of water (H_2O)!

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- Now look at ω_L , what do you notice ?

Nuclear Magnetic Resonance (NMR)

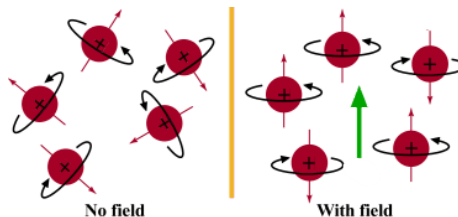
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- Why 1H ?
- Because the human body is composed of 50-65% of water (H_2O)!
- Now look at ω_L , what do you notice ?
- ω_L is in the RF range ! This is why the oscillations of B_1 are in the RF range.

Nuclear Magnetic Resonance (NMR)

To sum up:

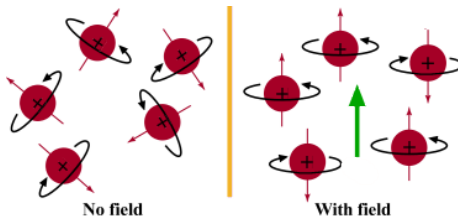
- Spinning hydrogens align with \mathbf{B}_0 and rotate around it at f_L
- We apply oscillations (\mathbf{B}_1) in the RF range which are absorbed by the hydrogen nuclei
- The frequency of these oscillations depends on the field strength B_0 and on the type of nucleus (γ)



Nuclear Magnetic Resonance (NMR)

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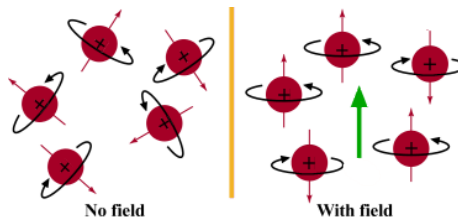
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Absorbed ??



Nuclear Magnetic Resonance (NMR)

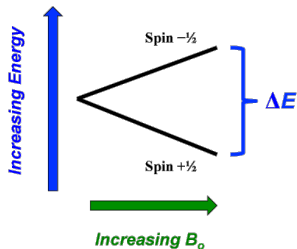


- A spin is at its lowest energy when it is aligned with B_0 and at its highest energy when it is opposite to B_0
- It seems that a 1H spin can only have two “pure” states in the presence of B_0 → Quantum Physics !!



Quantum Physics

- The spin of 1H is equal to $\frac{1}{2}$ and it can only have two possible *measurable* states (parallel or anti-parallel to \mathbf{B}_0).
- However, without \mathbf{B}_0 the two spin states are not measurable ! They are in a weighted superposition of both states
- **Zeeman effect:** $\Delta E = \gamma \frac{h}{2\pi} B_0$ where h is the Planck's constant and ΔE is the energy gap between two nuclear spin states
- **Planck–Einstein relation:** $\Delta E = hf$ The transfer of energy is quantized !





$$\Delta E = \gamma \frac{h}{2\pi} B_0 = hf \rightarrow f_L = \frac{\gamma B_0}{2\pi} \quad (4)$$

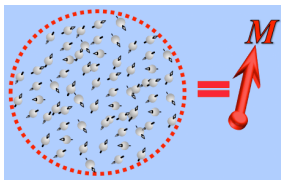
- This is another way to obtain the Larmor equation !
- So, the difference in energy between the two states is quantized and it depends on f_L
- This means that when a spin goes from its higher energy level to its lower energy level there is an emission of a photon, whose energy is hf_L
- And viceversa, an increase in energy results from absorption of a photon, whose energy must always be hf_L

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Net magnetization

The net magnetization vector \mathbf{M} is the sum of the magnetic moments (μ) of all spins in the sample. We can look at the sample as a whole, instead than looking at each spin !



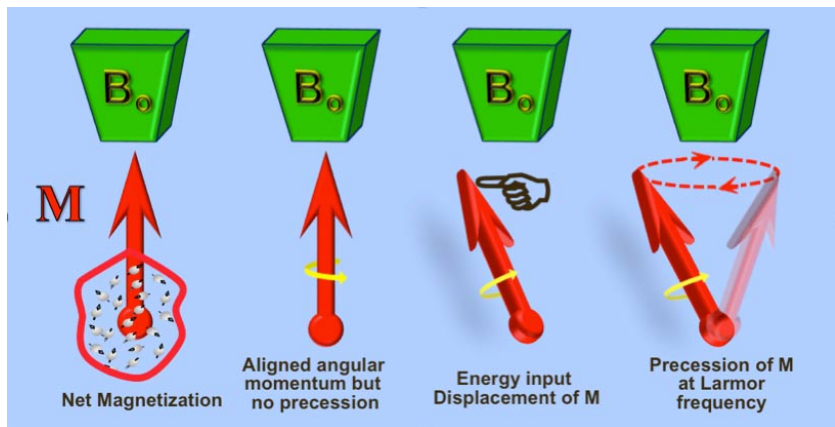
The maximum value of \mathbf{M} is:

$$M_0 \approx \frac{\gamma^2 h^2 B_0}{4kT} P_D \quad (5)$$

Thus, M_0 is directly proportional to P_D , the spin (proton) density, and to the magnetic field strength (B_0).

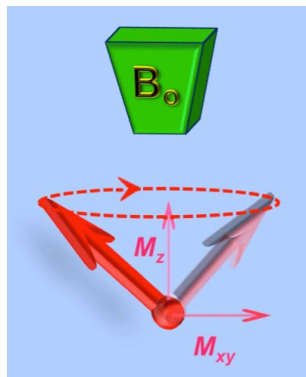
Net magnetization

- When we put a sample within B_0 , without any oscillation (i.e. no scanning), M is aligned with B_0 .
- When we apply a precise RF pulse at f_L (called B_1 and orthogonal to B_0), all the spins of 1H will start to precess, and so will do M !



Net magnetization

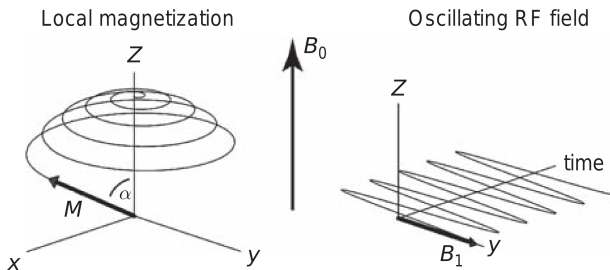
- The direction of \mathbf{B}_0 is commonly designated as the z-axis in a (stationary) Cartesian coordinate system placed on the top of the scanner.
- \mathbf{M} has thus a longitudinal component (M_z) and a transverse component M_{xy} , which are time-dependent



- Remember that as long as a static magnetic field \mathbf{B}_0 is present (even the one of earth), spins are always precessing !! This is true even after a RF-pulse \mathbf{B}_1 .
- However, when \mathbf{M} is aligned with \mathbf{B}_0 , it does not precess !!

Net magnetization

To sum up:



- Apply an oscillating RF pulse \mathbf{B}_1 orthogonal to \mathbf{B}_0 at f_L
- \mathbf{M} starts to rotate around \mathbf{B}_0 (the z-axis) tracing a spiral
- \mathbf{M} is tipped away from the z-axis of a flip angle α , which mainly depends on the strength of \mathbf{B}_1 and on the duration of the RF-pulse

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Relaxation

Relaxation is the process by which any excited \mathbf{M} relaxes back to its equilibrium state, namely parallel to \mathbf{B}_0 . It can be divided into two separate processes:

- 1 Longitudinal relaxation (\mathbf{T}_1)
- 2 Transverse relaxation (\mathbf{T}_2)

Bloch equations - stationary reference frame

Starting from the Larmor precession equation

$$\frac{d\mathbf{L}}{dt} = \gamma(\mathbf{L}(t) \times \mathbf{B}_0(t)) \quad (6)$$

In 1946 Felix Bloch found this set of equation about the NMR signal:

$$\begin{aligned} \frac{dM_x(t)}{dt} &= \gamma(\mathbf{M}(t) \times \mathbf{B}_0(t))_x - \frac{M_x(t)}{T_2} \\ \frac{dM_y(t)}{dt} &= \gamma(\mathbf{M}(t) \times \mathbf{B}_0(t))_y - \frac{M_y(t)}{T_2} \\ \frac{dM_z(t)}{dt} &= \gamma(\mathbf{M}(t) \times \mathbf{B}_0(t))_z - \frac{M_z(t) - M_0}{T_1} \end{aligned} \quad (7)$$

This set of equations is in a stationary reference frame positioned on the top of the scanner.

- In the presence of a constant \mathbf{B}_0 along the z-axis, namely $\mathbf{B}_0(t) = (0, 0, B_0)$, M_z is constant.
- Calling $M_{xy} = M_x + iM_y$, it results
$$M_{xy}(t) = M_{xy}(0) \exp(-i\gamma B_0 t) = M_{xy}(0) [\cos(\omega t) - i \sin(\omega t)]$$
- The M_{xy} component thus rotates around the z-axis (\mathbf{B}_0) with an angular velocity equal to $\omega = \gamma B_0$

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What if we use a rotating frame of reference around the z-axis with the same angular velocity ω ?

M_{xy} would appear stationary with respect to this new reference frame !

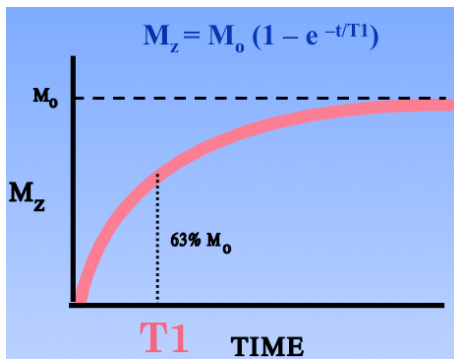
$$\begin{aligned} M'_z(t) &= M_z(t) \\ M'_{xy} &= \exp(i\omega t) M_{xy} \end{aligned} \tag{8}$$

Longitudinal relaxation (T_1)

In this new rotating reference frame, after applying an oscillating RF pulse \mathbf{B}_1 at f_L and with $\alpha = 90^\circ$, the Bloch equation for M'_z becomes:

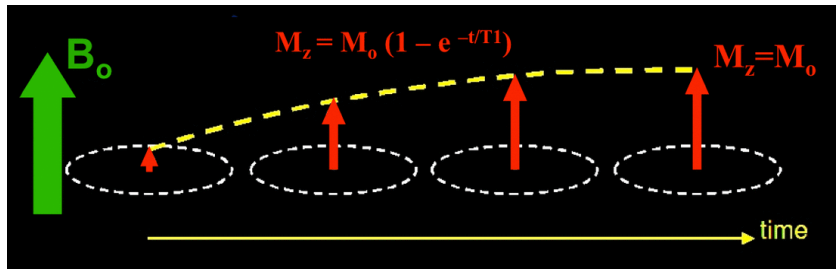
$$M'_z(t) = M_0 - (M_0 - M'_z(0)) \exp\left(-\frac{t}{T_1}\right) \quad (9)$$

If $\alpha = 90^\circ$ then $M'_z(0) = 0$:



Longitudinal relaxation (T_1)

- Longitudinal relaxation is also called *spin-lattice relaxation* because the spins return the energy they have absorbed by the RF pulse to the surrounding lattice (i.e. external environment)
- This energy is very small compared to normal molecular kinetic energy → almost unnoticed at room temperature
- T_1 depends on γ and on the mobility of the lattice, namely size and motion of the molecules of the sample → different tissues have different T_1 values

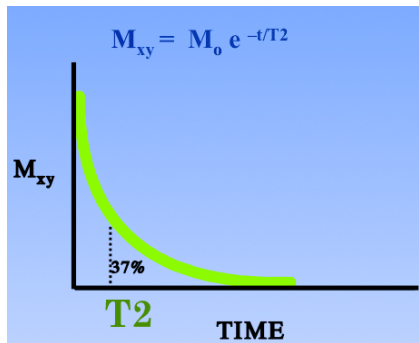


Transverse relaxation (T_2)

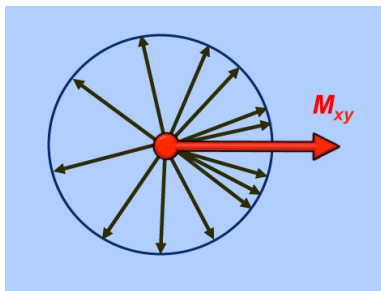
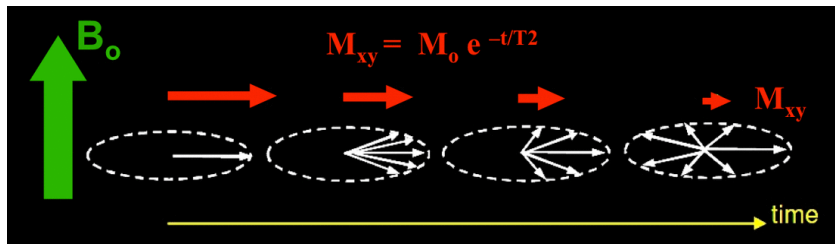
In the new rotating reference frame, after applying an oscillating RF pulse \mathbf{B}_1 at f_L and with $\alpha = 90^\circ$, the Bloch equation for M'_{xy} becomes:

$$M'_{xy}(t) = M'_{xy}(0) \exp\left(-\frac{t}{T_2}\right) \quad (10)$$

if $M'_{xy}(0) = M_0$



Transverse relaxation (T_2)



- In reality only the spins that are initially aligned with B_0 are in transverse phase coherence with one another after the 90° -pulse
- There is a (tiny) higher distribution of spins in the transverse plane parallel to M_{xy}

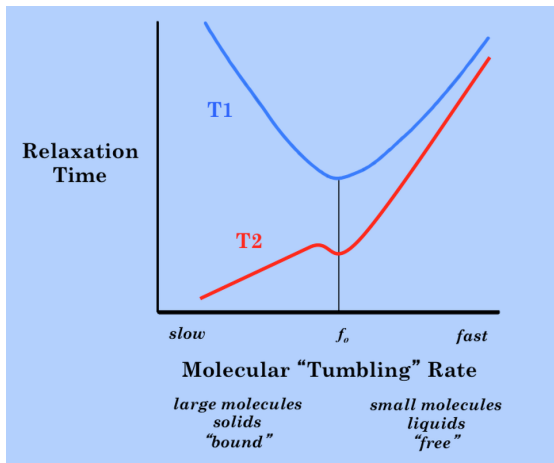
Relaxation

Tissue	T1 (msec)	T2 (msec)
Water/CSF	4000	2000
Gray matter	900	90
Muscle	900	50
Liver	500	40
Fat	250	70
Tendon	400	5
Proteins	250	0.1- 1.0
Ice	5000	0.001

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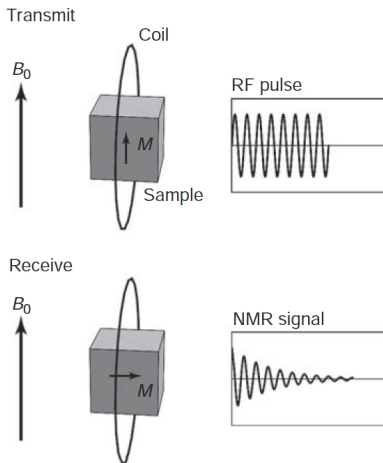
- T_1 is always longer than T_2
- Liquids have very long T_1 and T_2 values
- Dense solids (e.g. ice) have very short T_2 values

Relaxation



T1 and T2 as a function of molecular size and tumbling (movement) rate. The minimum value of T1 and dip in the T2 curve occurs when motion is at the Larmor frequency, $f_0 = f_L$.

Nuclear Magnetic Resonance (NMR)



- Certain atomic nuclei, such as the hydrogen, are able to absorb specific radio frequency energy and emit a small portion of that energy.
- The produced time-varying magnetic field will induce voltage in the coil (Maxwell–Faraday equation)
- This current, oscillating at the same frequency as the RF pulse, is the NMR signal

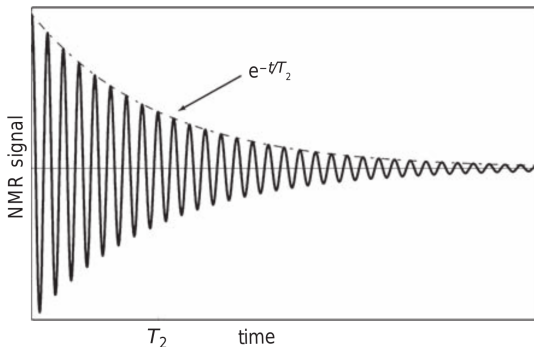
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Free induction decay (FID)

- The NMR signal is a small electrical current induced in the receiver coil by the precession of M_{xy} during resonance → **Faraday's Law of Induction** (a changing magnetic field induces a current in the coil)
- The resulting signal is called free induction decay (FID), which is a damped sine wave oscillating at the Larmor frequency (ω_L):

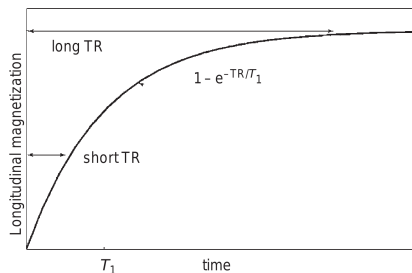
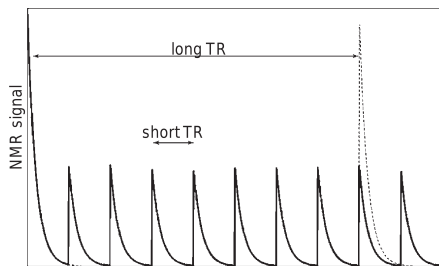
$$\sin(\omega_L t) \exp\left(-\frac{t}{T_2}\right)$$



- In a real NMR experiment M'_{xy} decays much faster than expected. This rate is called T_2^* where $T_2^* \leq T_2$
- T_2^* results principally from inhomogeneities in the main magnetic field. These inhomogeneities may be the result of intrinsic defects in the magnet itself or from susceptibility-induced field distortions produced by the tissue or other materials placed within the field

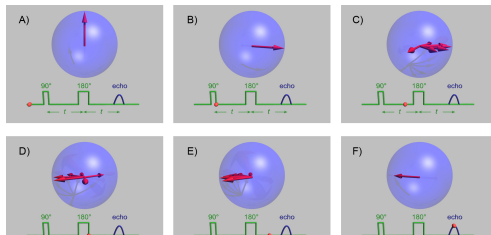
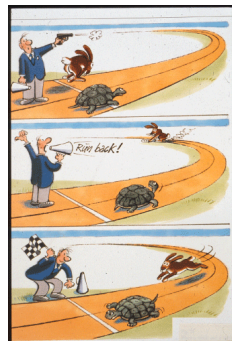
Repetition time (TR)

- TR : span of time between consecutive RF pulses (milliseconds)
- Every RF pulse generates a FID signal
- If the recovery of the first FID is not complete ($TR < T_1$), the next FID signal will be reduced

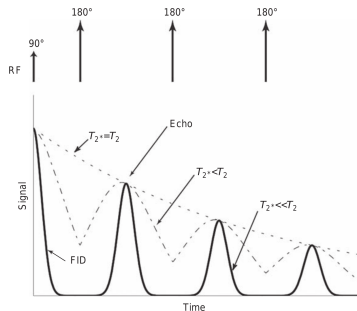
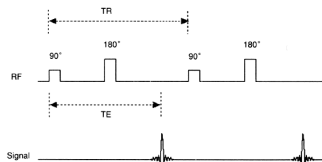


Spin echo - Echo Time (TE)

- A **spin echo** is produced by two successive RF-pulses (usually 90° and 180°) that create a detectable signal called the echo
- Due to inhomogeneities in the field, spins at different locations may precess at different rates
- After the 180° pulse (t) the slower spins are ahead and the faster ones are behind ! At $TE = 2t$ we have a complete refocusing that creates an echo. $T2$ can be correctly measured



Multi-echo pulse sequence

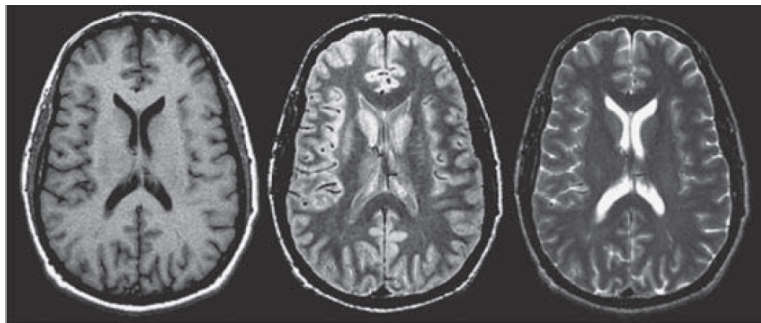


- Only the peak of the echo falls in the true T_2 decay curve
- Need a multi-echo pulse sequence using a series of 180° pulses

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Image contrast

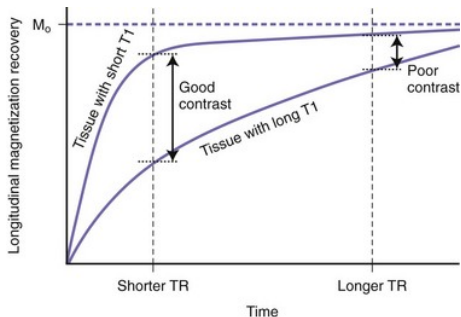


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Density-weighted
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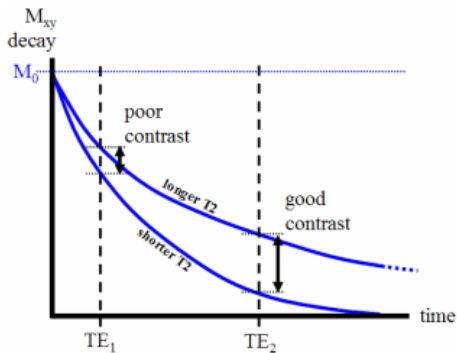
T_2 -weighted
(TR = 3800, TE = 102)

- Image contrast: signal difference between different tissues
- Tissues have different P_D , T_1 and T_2 . Signal intensity depends on these parameters
- Typically one aims to maximize the contrast between tissues focusing on (weighting) one of these parameters → Different TE and TR !!



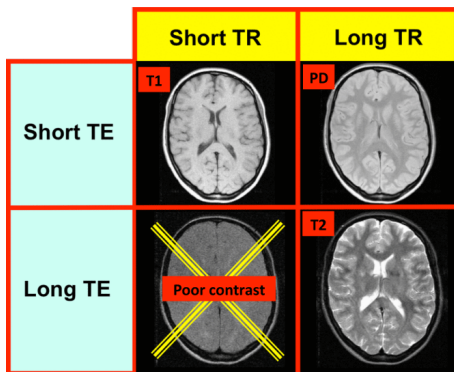
- Different tissues have different T_1
- If we use a long TR we have a poor contrast since M_z recovers completely for both tissues
- Need to use a short TR to have a good contrast

TE and T_2



- Different tissues have different T_2
- If we use a short TE we have a poor contrast but a high recorded NMR signal
- If we use a long TE we have a good contrast but a low recorded NMR signal

Image contrast



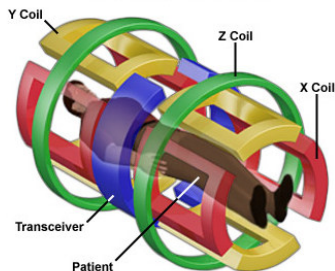
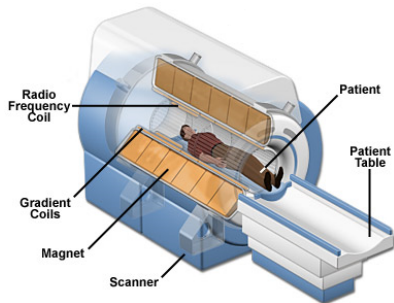
- P_D , T_1 and T_2 are positively correlated \rightarrow maximizing the sensitivity to all of them (short TR and long TE) leads to conflicting (opposite) contrast effects
- Contrast between gray and white matter is different ! Look at the ventricles

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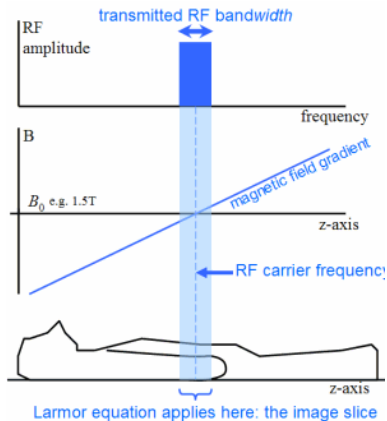
Magnetic gradients

- How do we localize the NMR signal ? 3 gradient coils
- Each gradient coil produces a magnetic field that varies *linearly* along an axis
- The three gradient coils produce magnetic gradients along 3 orthogonal directions → it can be produced a gradient along any direction !
- Gradients are added to B_0 and are much weaker



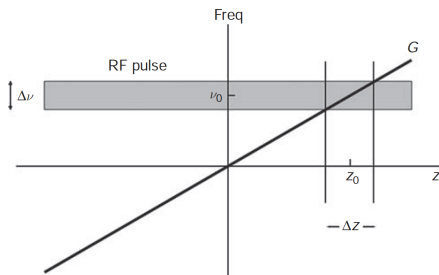
Slice selection

- We select a thin slice (1-2 mm thick) of the body
- We define a new coordinate system where the z-axis is perpendicular to the slice
- We produce a gradient field along the new z-axis ($\mathbf{B}(z) = \mathbf{B}_0 + z\mathbf{G}_z$) together with a tailored RF-pulse
- Its frequencies match the frequencies of the desired slice → only the protons within the chosen slice are excited !
- $f(z) = f_L + \gamma z G_z$



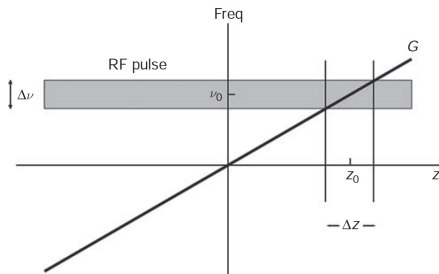
Slice selection

- $\Delta f = \gamma G_z \Delta z$: every slice has a finite width Δz which contains a range of frequencies (Δf) centered around f_L
- Δf is usually fixed (1-2 kHz), only variable is G_z (limit of the scan)
- $\Delta z = \frac{\Delta f}{\gamma G_z}$ strong gradients leads to small thickness
- We choose f_L , Δf and the RF-pulse (sinc pulse or Shinnar-Le Roux (SLR) algorithm)



Slice selection

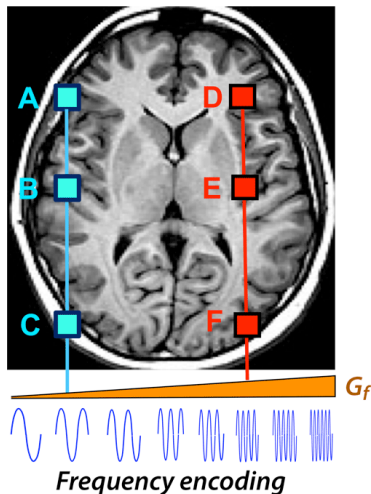
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NMR signal is the sum of all signals generated across the slice → **how to localize in the x-y plane ?**

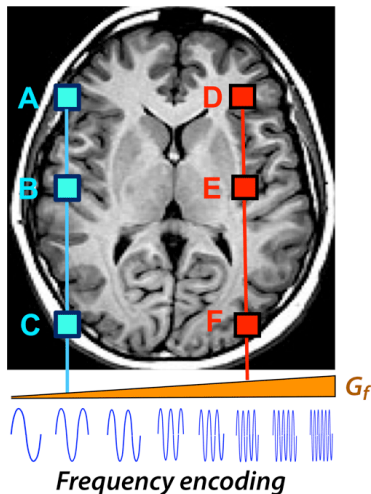
Frequency encoding

- Once selected the slice, we can use a frequency-encoding gradient (G_f) to specify one direction (i.e. x) within the slice
- $\mathbf{B}(x) = \mathbf{B}_0 + xG_f \rightarrow f(x) = f_L + \gamma x G_f$
- Pixels A,B,C,D,E,F in a static B_0 would resonate at the same f
- Thanks to G_f , A,B,C resonate at the same f which is lower than the one of D,E,F \rightarrow We can discriminate between the two columns !



Frequency encoding

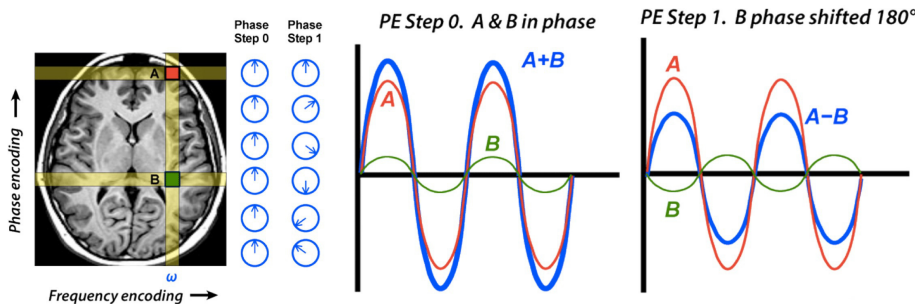
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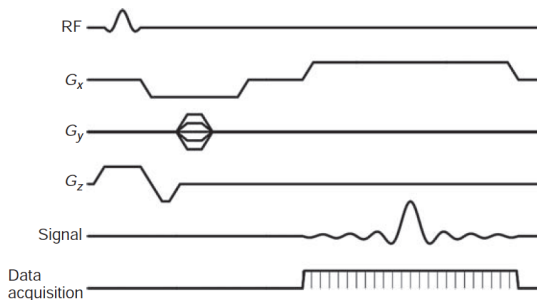
What about the other direction (i.e. y) ?

Phase encoding

- We use another gradient G_p for the last direction (i.e. y)
- We measure the sum of protons which all have the same frequency \rightarrow we use several measures with different phase shifts !
- If we focus on 2 pixels. At step 0: $S_0(t) = (I_A + I_B)e^{i\omega t}$
- At step 1, B has a 180° phase wrt A: $S_1(t) = (I_A - I_B)e^{i\omega t}$
- Algebra: $I_A = \frac{1}{2}S_0 + S_1$, $I_B = \frac{1}{2}S_0 - S_1$



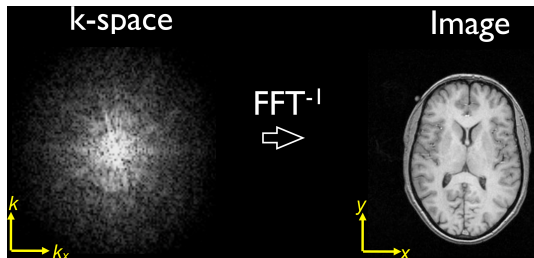
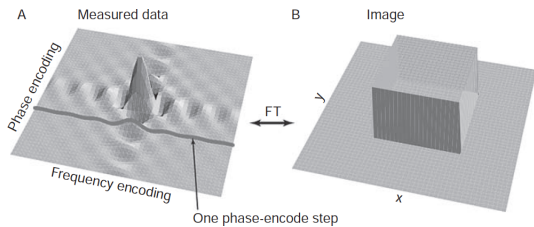
Spatial localization



- A RF pulse is first applied at a precise f_L together with a slice-selection gradient (z-axis)
- Two opposite G_f gradients are applied to produce an echo signal \rightarrow Each x is characterized by a different f
- Before the echo a G_p is kept for a time τ , protons will have a phase shift along the y -direction $\rightarrow \Delta\phi(y) = \gamma y G_p \tau$
- Given a G_f we need several different G_p (there are hundreds of pixels)

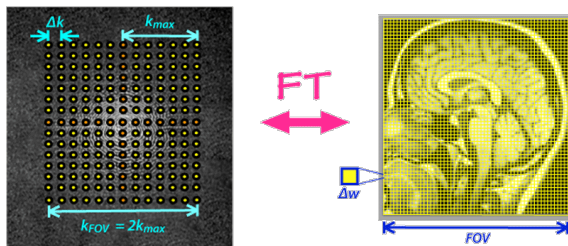
- We read the NMR signal while the x-gradient G_f is active. This means that at time t the protons will have a phase shift equal to $\Delta\phi(t) = \gamma G_f x t + \gamma y G_p \tau$ where $\gamma y G_p \tau$ is the phase shift already acquired due to G_p
- The NMR signal sums the contributions from all the locations, namely: $S(t) = \sum_x \sum_y I(x, y) \exp[i(\omega_L t + \gamma G_f x t + \gamma y G_p \tau)] \approx \sum_x \sum_y I(x, y) \exp[i(\gamma G_f x t + \gamma y G_p \tau)]$
- Calling $k_x = -(\gamma G_f t)$ and $k_y = -(\gamma G_p \tau) \rightarrow S(t) = \sum_x \sum_y I(x, y) \exp[-i2\pi(\frac{k_x}{2\pi}x + \frac{k_y}{2\pi}y)]$
- This is a spatial 2D Fourier transform ! It goes from the “k-space” of spatial frequency amplitudes ($S(k_x, k_y)$) to the “real space” of intensities ($I(x, y)$)

K-space



K-space

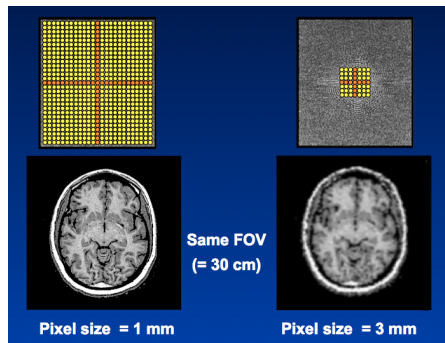
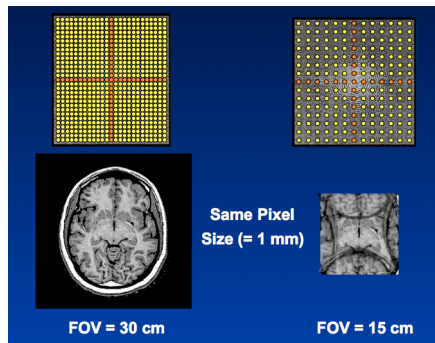
- Field of view (FOV): it can be seen as the size of the object of interest
- FOV and pixel width Δw (spatial resolution) are related to the sampling in the k-space (Δk) and to the dimension of the k-space
- Δk depends on the direction. One can increase the frequency encoding steps (x-axis, no time penalty) and the number of phase encoding steps (y-axis, time penalty) to reduce Δk . Usual k-space (matrix) size are 128x256 or 192x256.
- Δk also depends on the strength and duration of the gradient



$$\Delta k = 1 / \text{FOV}$$

$$\Delta w = 1 / k_{\text{FOV}}$$

K-space (Aliasing and Low resolution)



- Δk has doubled (more space between samples in the k-space)
- Same Δw (spatial resolution) but half FOV \rightarrow Aliasing
- Δk is the same
- Only certain spatial frequencies have been considered (reduced k_{FOV}) \rightarrow Δw has augmented, spatial resolution has decreased

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- We usually select and acquire one slice at a time or multi-slice using a composite RF-pulse to stimulate multiple slices. In both cases, we need to wait for full relaxation between RF-pulses → only long TR (i.e. T2-weighted)
- **To accelerate:** using symmetry of Fourier plane, full volume with 2 phase encoding, use smaller flip angles, etc
- **To increase SNR:** greater B_0 , greater voxel volume and slice thickness (more spins), acquisition time (average of repetitions of pulse sequence), increase FOV, decrease k-space size, decrease TE , increase TR

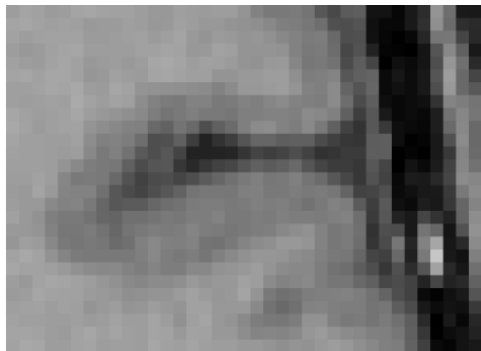
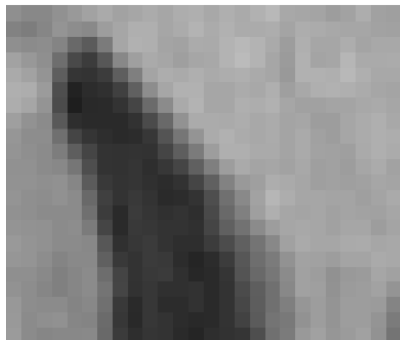
- T_1 : longitudinal relaxation time
- T_2 : transverse relaxation time
- P_D : proton density
- field heterogeneity
- physiological motion

- External magnetic fields
- Sequence parameters:
 - TR : repetition time
 - TE : echo time
 - flip angle
 - number of slices
 - FOV
 - slice thickness
 - slice orientation
 - gradient parameters
 - type of coils

Summary

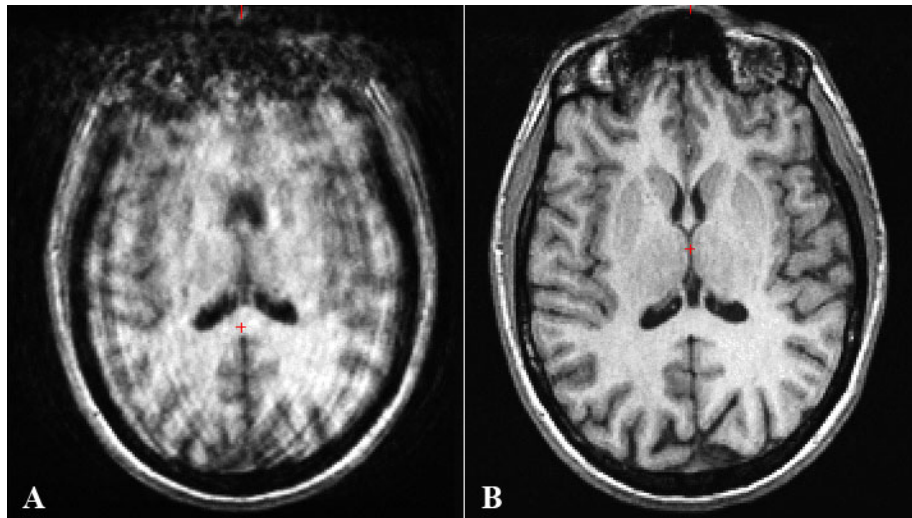
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Partial volume effect



Several tissue are averaged together in a single pixel.
Need for smaller pixels (higher resolution).

Motion artifact



Magnetic susceptibility artifacts

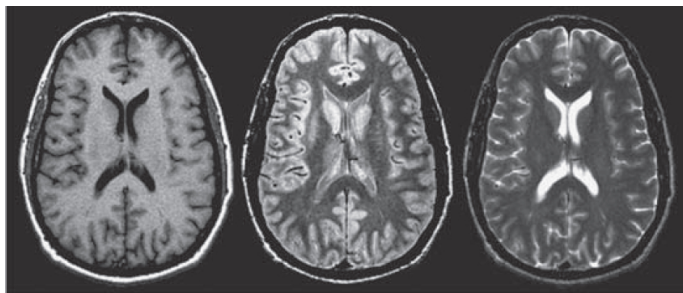


Due to metal implants, characterized by geometric distortion, very dark and very bright areas

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Anatomical (structural) MRI - Brain



T_1 -weighted
(TR = 600, TE = 11)

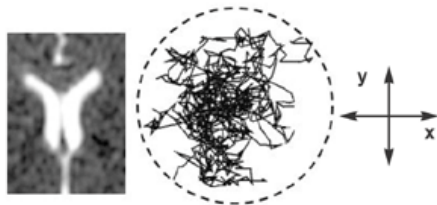
Density-weighted
(TR = 3000, TE = 17)

T_2 -weighted
(TR = 3800, TE = 102)

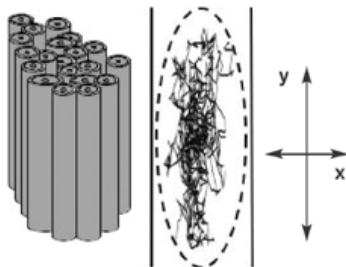
- It provides information to qualitatively and quantitatively describe the shape, size, and integrity of gray and white matter structures
- T1-w: good contrast between gray matter (GM) and white matter (WM). No for CSF
- T2-w: good contrast between CSF and brain tissue. Sensitive to subtle white matter alterations

Diffusion weighted MRI

A. Isotropic Diffusion

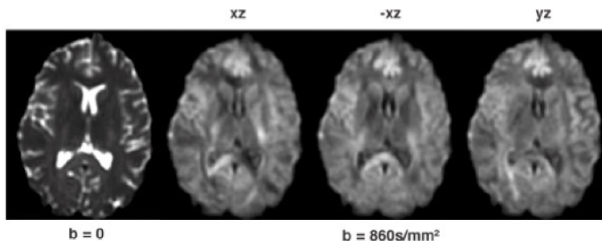


B. Anisotropic Diffusion



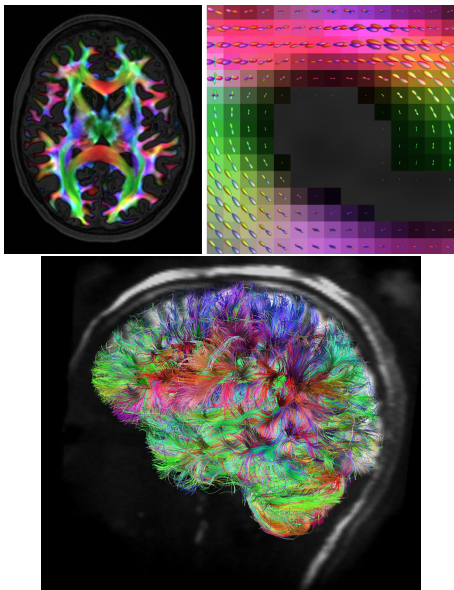
- DWI detects water diffusion → Brownian, unconstrained movement in large spaces (i.e. ventricles) produces isotropic diffusion. Constrained motion along a direction (i.e. white matter axons) produces anisotropic diffusion

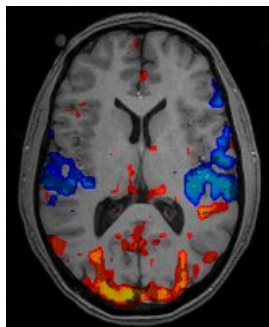
Diffusion weighted MRI



- Several diffusion gradients are applied (usually along 50-60 different directions) to detect water diffusion
- For each direction we use two opposite gradients that change linearly in the space. If there is no movement, the two signals cancel out, otherwise their difference is proportional to the movement of the water molecules along the direction
- $\frac{S_b}{S_0} = \exp(-b \cdot ADC)$, b is related to the strength of the gradient, ADC is the *apparent diffusion coefficient* since water is not “free” in biological tissues

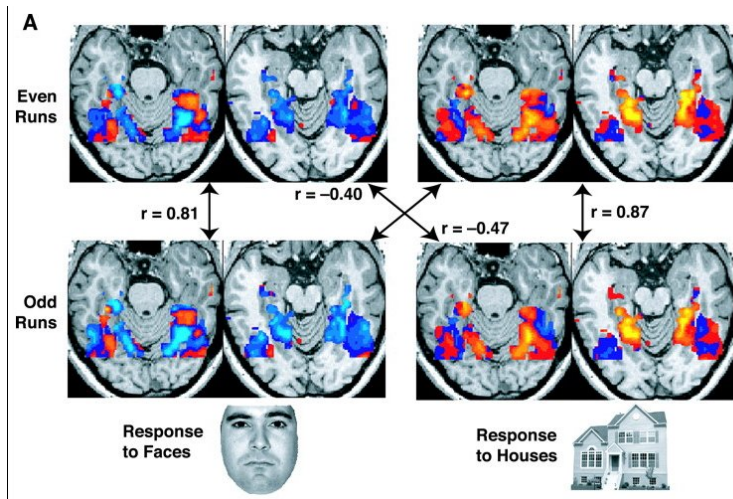
Diffusion weighted MRI





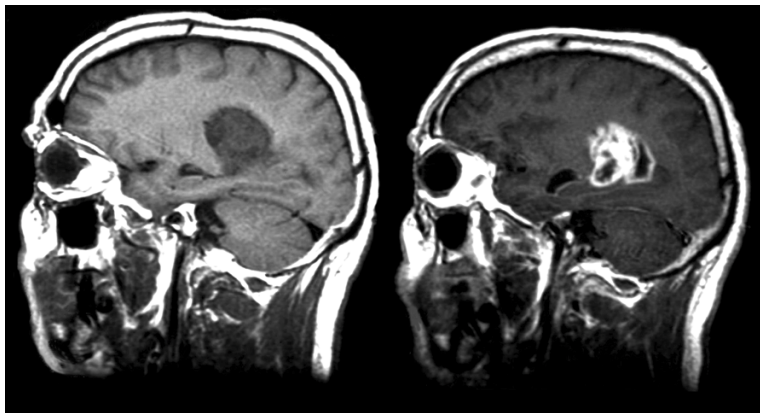
- First fMRI: blood-oxygen-level dependent (BOLD) contrast
- It measures brain activity by detecting hemodynamic responses → the more the brain is active in an area, the more the blood flows
- Hemodynamic response: blood releases oxygen to active neurons at a greater rate than to inactive neurons. Change in oxygenated and deoxygenated blood. Detectable with MRI since different magnetic susceptibility.

Functional MRI - Brain



- It is usually used for research purpose
- Detect signal changes in response to different stimuli

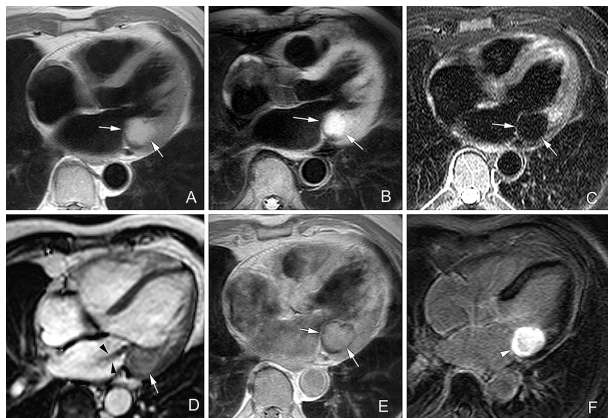
MRI contrast agent



- Contrast media used to improve the visibility of healthy or pathological tissues (i.e. tumor)
- The most common are the gadolinium-based



- Technique used to image blood vessels (e.g. arteries and veins)
- Usually based on contrast agents and short-TR sequences



- Non-invasive assessment of the function and structure of the cardiovascular system
- Coupled with diffusion imaging *VIDEO*

- ① Richard B. Buxton (2002). *Introduction to Functional Magnetic Resonance Imaging: Principles and Techniques*. Cambridge University Press
- ② F. Bloch. *Nuclear Induction*. Phys. Rev. 70 - 1946
- ③ <http://mriquestions.com/index.html>