FUZZY SPATIAL RELATIONS FOR HIGH RESOLUTION REMOTE SENSING IMAGE ANALYSIS: THE CASE OF "TO GO ACROSS"

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ABSTRACT

High resolution remote sensing (HR RS) images allow discriminating between different objects in a scene. Spatial reasoning techniques can be used to interpret and describe the scene. One component of spatial reasoning deals with the modeling and assesment of spatial relations among objects. In this work we propose three models that seize the semantics of the spatial relations "to go across" and "to go through" between a linear object and a region. To develop these three models we considered the usual perception of these natural language expressions which leads to the development of the three fuzzy models. They have been implemented and tested in scenes of HR RS images. Results are in good agreement with intuition.

***Index Terms*—** spatial relations, go across, natural language interpretation

1. INTRODUCTION

Spatial reasoning can be defined as the domain of spatial knowledge representation, in particular spatial relations between spatial entities [1], and of reasoning on these entities and relations. Among the possible applications of spatial reasoning are scene recognition, description and interpretation.

The presence of linear structures belonging to river or transport networks and spatial regions such as cities, agricultural fields, water surfaces, etc. on satellite images make spatial relations between a linear object and a surface very frequent and of prime importance to guide the interpretation of such images. Some examples of these relations are "to go across", "to go through", "to bypass", "to intersect", "to go along", "to enter", "to go into", "to surround", etc. In this work we concentrate on the binary spatial relation "to go across" between a reference region and a linear object.

Relations between linear objects and regions have been studied in the framework of spatial relations for GIS [2, 3, 4] and in the spatial cognition community [5, 6]. Egenhofer et al. [2] focused on the topological properties of the relations (observing the intersection between the boundaries, the interior and exterior of the region and the line, where the line boundaries are defined as its endpoints and the interior is every other point of the line). However, these relations were not directly linked to natural-language relations and did not completely match their usual semantical meaning. In [3] the link between topological constraints and natural language expressions was studied, showing that topology alone was not sufficient to define the relations between a linear object and a region. Shariff et al. [4] incorporated metrical measures in the topological models and presented crisp models for 59 natural language expressions. These models were calibrated using natural language expressions. One must note that natural language expressions between a linear object and a region depend on the shape of the region and their definition can be sometimes vague or imprecise. For instance the relation "to go across" can have the following meaning: a line goes across a region if it goes from one side of the region to the opposite side. When the region has a complex shape, the notion of opposite side becomes vague. Thus, the fuzzy sets framework is appropriate for modeling these relations since it captures the imprecision inherent to the spatial information and to the semantics of the relations. The models presented in this paper are developed within the fuzzy sets framework and can be easily extended to fuzzy objects.

The remainder of the paper is organized as follows. In Sec. 2, we briefly explain a human subject test that we performed to determine the meaning of the spatial relation "to go across". In Sec. 3, we present the proposed models obtained from the results of this test. In Sec. 4, we show some experimental results of the proposed models.

2. UNDERSTANDING THE RELATION

To understand the usual perception of the considered relation, 8 line-region configurations were proposed to 32 persons. Each configuration had a different region shape and a different linear trajectory. The persons were asked whether or not they agreed with the statement "the line goes across the region". Some space was left for commentaries. The obtained results were very different across the subjects, showing that it

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is difficult to obtain a consensus, and that it is more appropriate to evaluate the relation as a matter of degree rather than a crisp relation. To analyze the answers we observed each configuration separately, taking into account the commentaries made by the subjects. A first meaning derived from this analysis is very permissive and only considers the fact that a line enters or starts at the border of the region and leaves or ends at the border of the region. Two more restrictive meanings were also considered:

(i) the line goes from one side of the region to the opposite one,
(ii) the line goes deeply inside the region and passes close to its middle point.

Taking into account the remarks of [6, 4] the relation "to go through" and "to go across" are very similar, and in some dictionaries can be found as synonymous. But "go through" relies on topological aspects, since it refers to entering and leaving a medium [6], while "go across" has a more geometrical component: there is a specification regarding the way it should enter or leave the interior of the region. Therefore we are going to call the first model "to go through". The other two meanings will be called "to go across (i)" and "to go across (ii)", which are particular cases of the "to go through" relation.

3. LINE REGION RELATIONS

In the following, L denotes a linear object and R a region. L is defined as an elongated structure, and can be simply represented by its skeleton. We differentiate the end points of L from the rest. Using the same notation as in [2] the end points of L are denoted by L_a and L_b. The interior, boundary and exterior of a region R are denoted by R°, δR and R^c.

We make use of fuzzy conjunctions and disjunctions like t-norms and t-conorms, that are noted by t and T, respectively [7]. A fuzzy intersection relation measure denoted by μ_{int} and fuzzy non-intersection relation measure denoted by $\mu_{\neg int}$ are also involved in the proposed definitions [8].

3.1. To go through

This model is based on topological constraints. It verifies that there is an intersection between the interior of the region and the linear object, and that the linear object starts and ends outside the region. This definition is similar to the one proposed by Mark et al. [3], the only difference is that we admit that the extremities of the linear object are on the edge of the region.

Definition 3.1. Let L be a linear object with extremities L_a and L_b and let R be a region. Then, the degree to which L "goes through" R is given by:

$$\mu_{gothrough}(L,R) = t\left(\mu_{int}(L,R^\circ), \mu_{\neg int}(L_a \cup L_b, R^\circ)\right). \quad (1)$$

3.2. To go across (i)

For the "go across (i)" relation the "go through" relation should be satisfied and in addition the linear object should go from one side to the other.

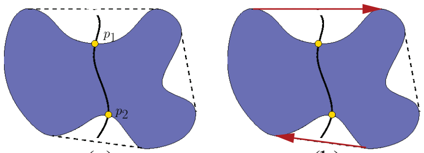
Studies in the area of spatial cognition suggest some definitions for what is meant by opposite sides. For instance, Talmy [6] states that the notion of opposite sides of a region is defined only when the region is ribbonal and the opposite sides are the two parallel sides that define the region. Another definition is based on the studies of Williams cited in [5], where an adult considers that a linear object goes across a region, if it goes through two different segments of the boundary region. These two segments can be the consecutive sides of a rectangle or two segments of the boundary of a circle. The second approach is more permissive than the one suggested by Talmy, and it is well adapted to regions with different shapes.

Considering the remarks of Williams we can define the degree to which two points p_1 and p_2 on the boundary of R are on opposite sides, by evaluating the degree to which the orientations of the two tangent vectors of the boundary at p_1 and p_2 are different. Unfortunately, this idea only gives good results if the region is convex. When objects have concavities we use the convex hull of the object to determine if the line goes from one side to the other. The convex hull of a region can be seen as the intersection of the minimum set of half planes that include the object. If p_1 or p_2 are in a concavity, then there must exist a plane which closes that concavity. We use the orientation of that plane, instead of the orientation of the tangent at the point to determine if the points are on opposite sides. We refer to this plane as the corresponding plane of the point. First, given a point $p \in \delta R$ we should find which is/are the corresponding planes of p.

Definition 3.2. Let $CH(R)$ be the convex hull of R, represented as a polygon with vertices $V = \{C_0, \dots, C_{n-1}\}$ and edges $E = \{(C_i, C_{i+1}) | i = 0, \dots, n-2\} \cup (C_{n-1}, C_0)$, where $C_i \in \delta R$ for $i = 0, \dots, n-1$. Let $p \in \delta R$. The corresponding planes of p are defined as:

(i) If $p \in V$, then there exists $i \in [0, n-1]$ such that $p = C_i$. And the corresponding planes are (C_{i-1}, C_i) and (C_i, C_{i+1}) where $i-1$ and $i+1$ are defined modulo n.
(ii) If $p \notin V$. Let $\phi(t) : [a,b] \rightarrow \mathbb{R}^2$ be a parametric function that describes the boundary of R, such that $\phi(a) = C_0$ and $\phi(b) = C_0$. Then there exists $s \in]a,b[$ such that $\phi(s) = p$. Let $C_i = \min_{C_i \in V} \{|s - t| : t < s \text{ and } \phi(t) = C_i\}$. The corresponding plane is (C_i, C_{i+1}) where $i+1$ is defined modulo n.

Once we have determined which are the corresponding planes of the points p_1 and p_2 we can establish if they are on opposite sides (See Fig.1).



(a) (b)

Fig. 1. (a) Entering and exiting points (p_1 and p_2), and convex hull of R in dotted lines. (b) Arrows showing the orientations of the corresponding planes of the points p_1 and p_2.

Definition 3.3. Let $p_1, p_2 \in \delta R$, and θ_1, θ_2 be the orientation of the corresponding planes of p_1 and p_2 respectively. The degree to which p_1 and p_2 belong to opposite sides is given by

$$\mu_{\neg side}(p_1, p_2) = f(|\theta_1 - \theta_2|), \quad (2)$$

where f is a continuous function such that $f : [0, 2\pi] \rightarrow [0, 1]$, $f(0) = f(2\pi) = 0$ and $f(\pi) = 1$. If p_1 or p_2 have more than one corresponding plane then we apply Eq. 2 to all the combinations of the corresponding planes and aggregate the answers with a disjunctive operator.

Thus, we can assess if "L goes across R" using the following definition.

Definition 3.4. The degree of satisfaction of the relation "L goes across R" using the meaning of going from one side to the opposite one is defined as:

$$\mu_{goacross1} = t\left(\mu_{gothrough}(L, R), \mu_{\neg sides}(p_1, p_2)\right), \quad (3)$$

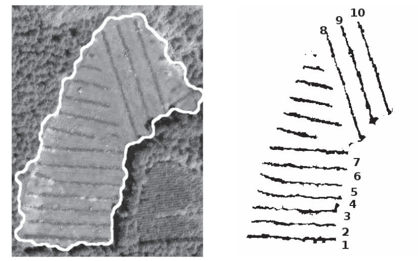
where p_1,p_2 are the entering and exiting points.

It is important to take into account the remark of Landau [5] in the case the region has a significant linear elongation. Then the sides are distinguished from the ends, and the relation will only involve the sides. If the linear object goes from one end to the other end of the region, then it will be more appropriate to describe the relation as going along.

3.3. To go across(ii)

A first approach to the idea of going deeply into a region is to think of passing near the center of mass of the region, like in [9]. A problem with this approach is that the center of mass can be outside the region. Hence, it is more appropriate to perceive how deeply a line goes into a region by observing how far it is from the boundary. If we create the distance map to the complement of the region, then the point which has the maximum value in the distance map is the deepest point of the region. Therefore, it is better to consider the Ultimate Erosion Points (UEP) rather than the center of mass. The UEP are defined as the regional maxima of the distance map in the interior of the region. To determine how deeply a line goes into a region we will measure how close it passes to one UEP.

First, we define a measure to determine whether L goes deep into R.



(a) (b)

Fig. 2. (a) Original image and region. (b) Ten quasi-linear paths.

Definition 3.5. Let $S = \{p_1, \ldots, p_n\}$ be the set of UEPs of R. Let $Vor(S)$ be the Voronoi partition associated to S. Let J be the set of indices of the voronoi polygons that are intersected by the L. Let $distMap$ be the distance map defined in the interior of the region, where for each $p \in R$, $distMap(p) = dist(p, R^c)$. Then the degree to which L goes deeply into R is given by:

$$\mu_{deep}(L, R) = \max_{j \in J} \left[g\left(\frac{distMap(p_j) - dist(L, p_j)}{distMap(p_j)}\right) . g\left(\frac{distMap(p_j)}{M_{max}}\right)\right], \quad (4)$$

where $M_{max} = \max_{p_i \in S} distMap(p)$ and g is an increasing function from $[0, 1]$ to $[0, 1]$.

Equation 4 is composed of two terms. The first term indicates how close does the line pass to a UEP and the second term measures if the chosen UEP is close to the deepest point of the region. Now we define the relation "L goes across R" using the following definition:

Definition 3.6. The degree of satisfaction of the relation "L goes across R" using the meaning of going deeply into the region is defined as:

$$\mu_{goacross2} = t\left(\mu_{gothrough}(L, R), \mu_{deep}(L, R)\right). \quad (5)$$

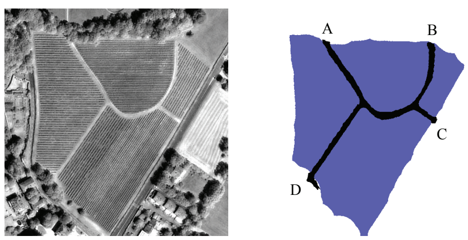
To avoid situations where the line goes deeply into the region and returns basically using the same path we can combine in a conjunctive way Eq. 5 with Eq. 2, or measure that the angle made between the entering point, the closest UE and the leaving point is not acute.

4. RESULTS

The above presented relations were evaluated in two cases, one for ten different paths and a region (Fig.2) and the other one for 6 paths and a region (Fig.3, where paths are named using their starting and ending points). The results are presented in Table 1 and in Table 2, respectively. In both cases, we can

Path	Degree *to go through*	Degree *to go deep*	Degree *to go across(i)*	Degree *to go across(ii)*
1	1.00	0.14	0.92	0.14
2	1.00	0.53	1.00	0.53
3	1.00	0.96	1.00	0.96
4	0.00	1.00	0.00	0.00
5	1.00	1.00	1.00	1.00
6	1.00	1.00	1.00	1.00
7	1.00	1.00	1.00	1.00
8	1.00	1.00	1.00	1.00
9	1.00	0.97	0.97	0.97
10	1.00	0.50	0.97	0.50

Table 1. Results for Fig.2 for the two interpretations of the "to go across" relation and the relations involved in their definition.



(a) (b)

Fig. 3. (a) Original image. (b) Paths and region.

observe that the obtained results agree with the perception of the relations. The relation "to go through" verifies the intersection with the interior and the boundaries, and the greater degrees were obtained for the paths that started and ended at the border of the region. Higher values were obtained for the degree of satisfaction of the relation "to go deep into" as the paths went profoundly into the region (paths $4-8$ in Fig.2 and all the paths in Fig.3 except path BC). The notion of opposite sides fits with the intuition. For points situated on almost parallel edges high values were obtained. The results reflect the need of using two different definitions for the relation "to go across" since in ambiguous cases (for example paths 1 and 10 in Fig.2 or path AB in Fig.3) it is not possible to reach a consensus.

Path	Degree *to go through*	Degree *to go deep*	Degree *to go across(i)*	Degree *to go across(ii)*
Path AB	1.00	1.00	0.00	1.00
Path AC	1.00	1.00	0.86	1.00
Path AD	1.00	1.00	0.95	1.00
Path BC	1.00	0.53	0.86	0.53
Path BD	1.00	1.00	0.95	1.00
Path CD	1.00	1.00	1.00	1.00

Table 2. Results for Fig.3 for the two interpretations of the "to go across" relation and the relations involved in their definition.

5. CONCLUSIONS AND PERSPECTIVES

We proposed three fuzzy models to represent the natural language relations of "to go across" and "to go through". These models are based on the results of a human subject test, therefore they consider the perception of the relation. The results of applying these models were shown experimentally, and highlighted the difficulty of finding a general model for the relation "to go across".

From this work, it is possible to develop other models for other relations which are related to the relations "to go across" or "to go through". For instance the relation "avoids" or "to enter".

6. REFERENCES

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