Approximate Parallelism between Fuzzy Objects: Some Definitions

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Abstract. Assessing the parallelism between objects is an important issue when considering man-made objects such as buildings, roads, etc. In this paper, we address this problem in the fuzzy set framework and define novel approximate parallelism notions, for fuzzy segments and non-linear objects or groups of objects. The proposed definitions are in agreement with the intuitive perception of this spatial relation, as illustrated on real objects from satellite images.

1 Introduction

We discuss the problem of defining parallelism between objects and fuzzy objects. This work is motivated by the importance of this spatial relation for describing human made objects such as buildings, roads, railways, observed in satellite images. Parallelism has been widely studied in the computer vision community in the perceptual organization domain, since it is an important feature of the grouping principles of the Gestalt theory [4].

Parallelism between linear segments was studied in several works, for example [7,6,5,8]. In [7] the parallelism is detected by assigning a significance value to determine that the detected parallelism has not been accidentally originated. A fuzzy approach is proposed in [6,5], leading to a measure of the degree of parallelism between two linear segments. The parallelism between curves was studied in [4,9], where it was treated as a shape matching problem.

The previous works focus on parallelism between crisp segments. We propose a definition to evaluate parallelism between fuzzy segments. Then we extend it to non-linear objects and groups of objects (crisp or fuzzy) taking into account the semantic meaning of the relation. The properties of our definitions are different from those desired in perceptual organization and are adapted to our purpose.

This paper is organized as follows. Section 2 contains considerations that should be taken into account when modeling the parallel spatial relation. A model for fuzzy segments is proposed in Sec. 3 and for non-linear objects in Sec. 4. Experimental results are shown in Sec. 5.

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2 Considerations for Modeling Parallelism

Parallelism can be of interest in multiple situations, between objects or their boundaries, between lines, between groups of objects, etc.

For linear segments to be parallel, we expect a constant distance between them, or that they have the same normal vectors and the same orientation. Although classical parallelism in Euclidean geometry is a symmetric and transitive relation, these properties are subject to discussion when dealing with image segments of finite length. When segments have different extensions as in Fig. 1(a), where B can be the boundary of a car, and A the boundary of a road, the symmetry becomes questionable. The statement "B is parallel to A" can be considered as true, since from every point in B it is possible to see (in the normal direction) a point of A, and the normal vectors at both points are the same. On the contrary, the way we perceive "B parallel to A" will change depending on our position: from point d it is possible to see a point of B in the normal direction with the same normal vector, while this is not possible from point c. In both cases (symmetrical and non symmetrical ones) the transitivity is lost. For example, in Fig. 1(b) and 1(c) the statements "A is parallel to B" and "B is parallel to C" hold, but "A is not parallel to C" since it is not possible to see C from A in the normal direction to A. This example also illustrates the interest of considering the degree of satisfaction of the relation instead of a crisp answer (yes/no). Then the relation "B parallel to A" will have a higher degree than "A parallel to B" in Fig. 1(a).

Now, when considering objects, parallelism is often assessed visually for elongated objects, based on the portions of their boundaries that are facing each other. Figure 1(d) shows two configurations where the portions of the boundary of object $A1$ and of $A2$ that face B have the same length. Do we want to assign the same degree of satisfaction to the parallel relations in both situations? Therefore, the dimensions of the objects can also influence the way we perceive parallelism.

The parallel relation can also be considered between a group of objects $\{A_i\}$ and an object B, typically when the objects in the group are aligned and B is elongated. For example a group of boats and a deck in a port. When evaluating the relation "$\{A_i\}$ is parallel to B", actually we are evaluating that the whole set $\{A_i\}$ and the boundary of B that faces $\{A_i\}$ have a similar orientation, and that there is a large proportion of $\cup_i A_i$ that sees the boundary of B in the normal direction to the group. Similar considerations can be derived when considering

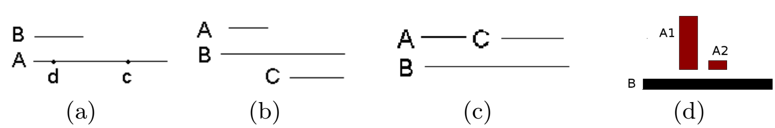


Fig. 1. (a),(b),(c) Examples where parallelism should preferably be considered as a matter of degree, and should not be necessarily symmetrical and transitive. (d) Example of parallelism between objects with different dimensions.

the relation "B is parallel to $\{A_i\}$" or the relation between two groups of objects. All these considerations form the basis for the formal models provided in the next sections.

Notations: Let $\mathcal{S}$ be the image space, and $\mathcal{F}$ the set of fuzzy sets defined over $\mathcal{S}$. Let A denote a fuzzy set, defined through its membership function $\mu_A : \mathcal{S} \to [0,1]$. Let $\boldsymbol{u}_{\theta_A}$ denote the normal unit vector to the principal axis of A, with angle θ_A with respect to the x-axis. Fuzzy conjunctions (t-norms) and disjunctions (t-conorms) are denoted by t and T respectively. In this work we make use of some definitions of fuzzy mathematical morphology and spatial relations, such as the directional dilation. The directional dilation of a fuzzy set μ in a direction $\boldsymbol{u}_\theta$ is defined as [3]:

$$D_{\nu_\theta}(\mu)(x) = \sup_y t[\mu(y), \nu_\theta(x-y)] \ , \tag{1}$$

where ν_θ is a fuzzy directional structuring element chosen so as to have high membership values in the direction $\boldsymbol{u}_\theta$ and its value at a point $x = (r, \alpha)$ (in polar coordinates) is a decreasing function of $|\theta - \alpha|$ modulo 2π (see Fig. 2(b)).

Another notion that will be useful is the admissibility of a segment: a segment $]a, b[$, with $a \in A$ and $b \in B$ (for A and B closed), is said to be admissible if it is included in $A^C \cap B^C$ [2]. In the fuzzy case, this extends to a degree of admissibility denoted by $\mu_{adm}(a, b)$.

3 Parallelism between Fuzzy Segments

In this section we propose a definition of parallelism between fuzzy boundaries or fuzzy lines, including the particular case of crisp linear segments, and taking into account the considerations of Sec. 2. The degree of satisfaction of the relation "A is parallel to B" should depend on the proportion of μ_A that sees μ_B in the normal direction of μ_A, and be high if the visible part of μ_B has a similar orientation to the one of μ_A. The degree to which a point $x \in \mu_A$ sees μ_B in the direction $\boldsymbol{u}_{\theta_A}$ is equivalent to the degree to which the point is seen by μ_B in the direction $\boldsymbol{u}_{\theta_A+\pi}$. To determine this degree we use the directional dilation (Eq. 1), which provides a fuzzy set, where the membership value of a point $x \in \mathcal{S}$ corresponds to the degree to which this point is visible from μ in the direction $\boldsymbol{u}_\theta$ [1,2].

Definition 1. *Let $\mu_A, \mu_B \in \mathcal{F}$. The subset of μ_A that sees μ_B in the direction $\boldsymbol{u}_{\theta_A}$ is denoted by $\mu^B_{A_\theta}$ and is equivalent to the intersection of μ_A and the fuzzy directional dilation of μ_B in direction $\boldsymbol{u}_{\theta_A+\pi}$. It has the following membership function:*

$$\forall x \in \mathcal{S}, \ \ \mu^B_{A_{\theta_A}}(x) = t[\mu_A(x), D_{\nu_{\theta_A+\pi}}(\mu_B)(x)] \ . \tag{2}$$

The set $\mu^B_{A_{\theta_A}}$ can be interpreted as the projection of μ_B onto μ_A. The proportion of μ_A that sees μ_B in the normal direction $\boldsymbol{u}_{\theta_A}$ is given by the relation $\mu_P(\mu_A, \mu_B)$ expressed as: $\mu_P(\mu_A, \mu_B) = |\mu_{A^B_{\theta_A}}|/|\mu_A|$.

(a) Original Segments (b) ν_{θ_A} (c) $D_{\nu_{\theta_A}}(\mu_A)$ (d) $\mu^B_{A_\theta}$ and $\mu^A_{B_{\theta_A+\pi}}$

Fig. 2. Illustration of the computation of parallelism between segments using directional dilation. Membership values vary from 0 (white) to 1 (black).

We have $\mu_P(\mu_A, \mu_B) = 1$ if and only if $\forall x \in S \ \mu^B_{A_{\theta_A}}(x) = \mu_A(x)$. This occurs when the projection of the segment μ_B onto μ_A is equal to μ_A.

In a similar way, we define the portion of μ_B visible from μ_A as $\forall x \in \mathcal{S}, \ \mu^A_{B_{\theta_A+\pi}}(x) = t(\mu_B(x), D_{\nu_{\theta_A}}(\mu_A)(x))$ (See Fig. 2).

Definition 2. *The relation "A is parallel to B" is given by the following measure:*

$$\mu_{\|N}(\mu_A, \mu_B) = t[\mu_P(\mu_A, \mu_B), \mu_\alpha(\mu^A_{B_{\theta_A+\pi}}, \mu_A)] \ , \tag{3}$$

where $\mu_\alpha(\mu, \mu')$ is a function that penalizes large orientation differences between the orientations of μ and μ', for example:

$$\mu_\alpha(\mu, \mu') = \begin{cases} 1 & \text{if } 0 \le |\theta_\mu - \theta_{\mu'}| < a, \\ (b - |\theta_\mu - \theta_{\mu'}|)/(b-a) & \text{if } a \le |\theta_\mu - \theta_{\mu'}| < b, \\ 0 & \text{if } b \le |\theta_\mu - \theta_{\mu'}| \end{cases} \tag{4}$$

In some contexts a symmetrical relation is needed (for example in perceptual organization), and is then expressed as "A and B are parallel". In such cases, we verify that each set is visible from the other in the normal direction and that the orientations of both sets are similar, leading to the following definition.

Definition 3. *The degree of satisfaction of the symmetrical relation, "A and B are parallel" is expressed by:*

$$\mu_{\|S}(\mu_A, \mu_B) = t[T[\mu_P(\mu_B, \mu_A), \mu_P(\mu_A, \mu_B)], \mu_\alpha(\mu^B_{A_{\theta_B+\pi}}, \mu_B), \mu_\alpha(\mu^A_{B_{\theta_A+\pi}}, \mu_A)], \tag{5}$$

Proposition 1. *Both relations (Definitions 2 and 3) are invariant with respect to geometric transformations (translation, rotation, scaling).*

None of the relations is transitive, as discussed in Sec. 2. But we have the following partial result in the crisp case:

Proposition 2. *Let A, B, C be linear crisp segments, if $\mu_{\|N}(A,B) = 1$, $\mu_{\|N}(B,C) = 1$ and $\theta_A = \theta_B = \theta_C$, then $\mu_{\|N}(A,C) = 1$.*

This result shows that in the crisp case we have transitivity. To have the transitivity property, it is necessary that $\theta_A = \theta_B = \theta_C$, since $\mu_\alpha(A,B) = 1$ and

$\mu_\alpha(B, C) = 1$ do not imply $\mu_\alpha(A, C) = 1$ due to the tolerance value a of the function μ_α (See Eq. 4). To have the transitivity without imposing the condition $\theta_A = \theta_B = \theta_C$, it is necessary that μ_α is a linear function (i.e $a = 0$). But, this is restrictive.

It is clear that both relations are reflexive. However, depending on the context we may not want to consider intersecting objects as parallel. In this case, it is necessary to combine in a conjunctive way the previous degree (Def. 2 or Def. 3) with a degree of non-intersection between the two sets.

4 Parallelism between Objects

As explained in Sec. 2, parallelism can occur between more than two objects. The following paragraphs detail each situation of interest.

Parallelism between Two Objects
For objects of similar spatial extension, we evaluate the relation between the boundaries that are facing each other. These boundaries correspond to the boundaries of the objects that delimit the region between the objects, and are defined as the extremities of the admissible segments [2]. We call this portion of the boundary, the admissible boundary. When the admissible boundary of each object can be approximated by one segment the degree of satisfaction of the relation is evaluated using one of the equations presented in Sec. 3.

For the case where the admissible boundary is approximated by several segments we concentrate on the non symmetric relation. A is considered parallel to B if for every segment of the admissible boundary of A there exists a segment of the admissible boundary of B that is parallel to it.

Definition 4. *Let A and B be two fuzzy sets, defined through their membership functions μ_A and μ_B. Let $\{\mu_{\delta A_i}\}_{i=0}^{I}$ and $\{\mu_{\delta B_j}\}_{j=0}^{J}$ be the approximation by fuzzy sets of the admissible boundary of μ_A with respect to μ_B, and vice-versa. The degree of satisfaction of the relation "A is parallel to B" is defined as:*

$$\mu_{\parallel N}(\mu_A, \mu_B) = \sum_i |\mu_{\delta A_i}| \max_j \mu_{\parallel N}(\mu_{\delta A_i}, \mu_{\delta B_j}) / |T(\mu_{\delta A_0}, \ldots, \mu_{\delta A_I})| \ . \quad (6)$$

The degree to which each $\mu_{\delta A_i}$ is parallel to $\mu_{\delta B}$ is equal to $\max_j \mu_{\parallel N}(\mu_{\delta A_i}, \mu_{\delta B_j})$. Then this degree is weighted by the importance of $\mu_{\delta A_i}$ in the admissible boundary of A.

To calculate the degree $\mu_{\parallel N}(\mu_{\delta A_i}, \mu_{\delta B_j})$ Eq. 2 can be used with a modification of $\mu^B_{A_{\theta_A}}$ to take into account potential hidden parts due to concavities or corners of the objects. A point $x \in \mu_{\delta A}$ will see a point $y \in \mathcal{S}$ in the direction $\boldsymbol{u}_{\theta_A}$ if it is visible according to Eq. 1 and also if the segment $]x, y[$ is admissible (with respect to μ_A and μ_B). This is expressed as:

$$\forall x \in \mathcal{S}, \ \ \tilde{\mu}^B_{A_{\theta_A}}(x) = t[\mu_A(x), D_{\nu_{\theta_A + \pi}}(\mu_B)(x), \mu_{adm}(]x, y[)] \ . \quad (7)$$

When objects have different spatial extensions the boundaries that should be considered are different if we want to take into account the dimension of the object (see Sec. 2). In this case, we can use the admissible or closest boundaries and/or include a term that expresses the relation between the principal axis of both objects.

A Ggroup of Objects Parallel to an Object

Let $\mathcal{A} = \{A_i\}_{i=0}^{I}$ be a finite set of fuzzy sets with membership functions μ_{A_i}. Let B be another fuzzy set with membership function μ_B.

For $\mathcal{A}$ to be parallel to B it is necessary that the objects of $\mathcal{A}$ are aligned. Considering each object of the group as a point (typically its center of mass), we can say that they are aligned if for every couple of points the orientation of the vector that joins them is equal to the orientation of the vector that joins the first and last points.

Definition 5. *Let m_i be the center of mass of each μ_{A_i}. Suppose that the set $\mathcal{A} = \{\mu_{A_i}\}_{i=0}^{I}$ is organized by a lexicographic order of its centers. Let μ_{align} be the relationship of alignment between fuzzy sets. This relationship is defined as:*

$$\mu_{align}(\mathcal{A}) = \min_{i<I} \mu_\alpha(T(\mu_{A_0}, \ldots, \mu_{A_I}), \{\mu_{A_i}(m_i), \mu_{A_{i+1}}(m_{i+1})\}), \tag{8}$$

where the set $\{\mu_{A_i}(m_i), \mu_{A_{i+1}}(m_{i+1})\}$ has two points and its central axis is the vector joining the two centers. The function $\mu_{\alpha'}$ has same shape as the function used in Eq. 4, and it penalizes large orientation differences.

The values of tolerance for $\mu_{\alpha'}$ can be different from those used for the parallel relation. This definition considers that objects have similar dimensions, and it does not take into account the distance between the objects.

To evaluate the degree of satisfaction of "$\mathcal{A}$ is parallel to B", we calculate for every i the portion of the closest boundary of μ_{A_i} to μ_B, which we denote $\mu_{\gamma A_i}$. And for μ_B we consider the linear boundary $\mu_{\gamma B}$ that is closest to the group.

Definition 6. *The degree of satisfaction of the relation "$\mathcal{A}$ is be parallel to B" is given by:*

$$\begin{aligned} \mu_{\|N}(\mathcal{A}, \mu_B) = t[&\mu_P(T(\mu_{\gamma A_0}, \ldots, \mu_{\gamma A_I}), \mu_{\gamma B}), \\ &\mu_\alpha(T(\mu_{A_0}, \ldots, \mu_{A_I}), \mu_{\gamma B}), \mu_{align}(\mathcal{A})]. \end{aligned} \tag{9}$$

An Object Parallel to a Group of Objects

Using the same notations as above, let us assume that the set $\{\mu_{A_i}\}_{i=0}^{I}$ is organized by a lexicographic order of its centers. Let $\beta_A \in \mathcal{F}$ be the region composed of the union of the regions between two consecutive elements of $\mathcal{A}$ (see [2]). For "B is parallel to $\mathcal{A}$" to be true, it is necessary that the objects in the group are aligned, that $\mu_{\gamma B}$ and the group of objects have a similar orientation and that there is a large proportion of $\mu_{\gamma B}$ that sees the group of objects or β_A:

Definition 7. *The degree of satisfaction of the relation "B is parallel to $\mathcal{A}$" is given by:*

$$\mu_{\|N}(\mu_B, \mathcal{A}) = t[\mu_P(\mu_{\gamma B}, \mu_{\gamma A'}), \mu_\alpha(T(\mu_{A_0}, \ldots, \mu_{A_I}), \mu_{\gamma B}), \mu_{align}(\mathcal{A})] \;, \tag{10}$$

where $\mu_{\gamma A'}$ denotes the admissible boundaries of $T(\mu_{A_0}, \ldots, \mu_{A_I}, \beta_A)$.

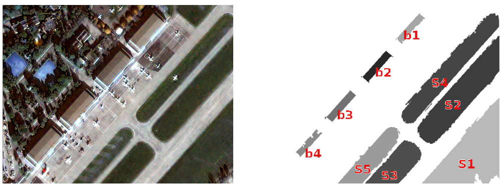
Using the same notation as in Def. 7, we can define the parallelism between two finite sets of fuzzy sets $\mathcal{A} = \{A_i\}_{i=0}^{I}$ and $\mathcal{B} = \{B_j\}_{j=0}^{J}$:

Definition 8. *The degree of satisfaction of the relation "$\mathcal{A}$ is parallel to $\mathcal{B}$" is given by:*

$$\begin{aligned}\mu_{\|N}(\mathcal{A},\mathcal{B}) = t[&\mu_P(T(\mu_{\gamma A_0},\dots\mu_{\gamma A_I}),\mu_{\gamma B'}),\mu_{align}(\mathcal{A}),\mu_{align}(\mathcal{B}),\\ &\mu_\alpha(T(\mu_{A_0},\dots,\mu_{A_I}),T(\mu_{B_0},\dots,\mu_{B_J}))]\ ,\end{aligned} \tag{11}$$

5 Results

We evaluated the parallel relation between two objects for the labeled objects of Figs. 3 and 4. For these examples we used $a = \pi/12$ and $b = \pi/6$ in Eq. 4.



A	B	$\mu_{\|\|N}(A,B)$	$\mu_{\|\|N}(B,A)$
b2	S4	0.94	0.55
b3	S5	0.97	0.87
b4	S5	0.89	0.66
S2	S4	0.97	0.97
S4	S1	0.87	0.94
S5	S3	0.90	0.95
S3	S1	0.78	0.43
b1	S4	0.90	0.69

Fig. 3. Original image, segmented image and results

To evaluate the relation between objects with different spatial extensions, we used the closest boundaries that had a similar orientation to the principal axis of the objects. From Fig. 3 we observe that the obtained results fit with the intuition. For objects that have similar spatial extensions ($S2$ and $S4$ or $S3$ and $S5$), similar values were obtained for $\mu_{||N}(A,B)$ and $\mu_{||N}(B,A)$. The results for $b2$ and $S4$, $b1$ and $S4$ and $b4$ and $S5$ reflect that when objects have different extensions the results are not symmetric.

A	B	$\mu_{\|\|N}(A,B)$	$\mu_{\|\|N}(B,A)$
R1	R2	0.93	0.85
R2	H1	0.14	0.29
R2	H2	0.42	0.69
R2	H3	0.22	0.74
R1	H3	0.00	0.38

Fig. 4. Original image, segmented image and results

Figure 4 shows a fuzzy segmentation of the image. Again the objects with similar spatial extension $R1$ and $R2$ have similar values for $\mu_{||N}(A,B)$ and $\mu_{||N}(B,A)$.

Our definitions for the parallelism between a group of objects and an object (Def. 7 and 6) were applied to the objects in Fig. 5. We used $a = \pi/18$ and $b = \pi/6$ in $\mu_{\alpha'}$ of Def. 5. As the boundaries involved in the relation of $B1$, $B2$ and $D1$ have similar spatial extensions the results are almost symmetrical, agreeing with intuition.

A	B	$\mu_{\|\|N}(A,B)$	$\mu_{\|\|N}(B,A)$
B1	D1	0.94	0.94
B2	D1	0.95	0.95
B1	B2	0.85	0.87

Fig. 5. Original image, segmented image and results

6 Conclusion

In this work we discussed the considerations that should be taken into account when modeling the parallel relation. We highlighted that the parallel relation depends on the situation and the context. We presented a definition of parallelism between two objects of similar spatial extensions, and briefly discussed the case of objects with different spatial extensions. Illustrations on real objects show the interest of the proposed definition.

Future work aims at extending the notion of alignment and parallelism to objects of different sizes and to more complex situations.

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