Representation of structural information in images using fuzzy set theory

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Abstract— **In this paper, we show how fuzzy set theory can be used to represent structural information in images, in particular, relationships between imprecise objects, defined as spatial fuzzy sets. We distinguish two types of relationships: on the one hand, relationships that are well defined in the case of crisp objects (like adjacency or distance), and on the other hand, relationships that do not find any consensual definition even in the binary case (typically relative position between objects). We propose several ways to generalize relationships of the first class in order to incorporate imprecision attached to the objects. For the second class, we argue that fuzzy definitions are appropriate even when dealing with crisp objects, and we propose original definitions.**

Keywords— **Fuzzy sets, image processing under imprecision, structural information, fuzzy spatial relationships.**

I. Introduction

The use of fuzzy sets in image processing has gained an increasing interest (see e.g. [11], [2]). Their power to deal with imprecise image information relies on:

- the ability of fuzzy sets to represent spatial information in images along with its imprecision, at different levels and under different forms (numerical as well as symbolic),
- the recent generalization of operations to fuzzy sets in order to manage spatial information,
- and information fusion using fuzzy combination operators, and leading to decision making.

We consider here fuzzy objects, i.e. fuzzy sets defined directly in the image space $\mathcal{S}$, through their membership function μ. For each image point $x \in \mathcal{S}$, $\mu(x)$ is a value in $[0,1]$ representing the membership of x to the fuzzy object defined by μ.

The main information contained in the images consists of properties of the objects and of relationships between objects, both being used for pattern recognition and scene interpretation purposes. Relationships between objects are particularly important since they carry structural information about the scene, by specifying the spatial arrangements between objects.

In this paper, we show how fuzzy set theory can be used to represent structural information in images, in the form of relationships between imprecisely defined objects, that are represented as spatial fuzzy sets.

We distinguish in Section II two types of relationships for representing structural information, the first one corresponding to relations that are well defined in the crisp case, and the second one to relations that are vague even in the crisp case. Then we show in Section III how to extend relationships of the first type to fuzzy objects, and illustrate this construction on two examples: adjacency and distances. In Section IV, we show that fuzzy concepts are powerful for defining relationships of the second type, even on crisp objects. The example of relative position is a good illustration. In Section V we illustrate these relationships on fuzzy objects obtained from 3D MR brain images.

II. Structural information as two types of spatial relationships

Model-based or case-based pattern recognition often relies on similarity measures, designed for comparing shapes and objects according to several aspects or features. Spatial information constitutes an important part of these features in image processing and scene interpretation, and it is either related to each object itself, or related to relationships between objects. We assume that the objects of interest have been segmented in a previous step. This segmentation may be coarse, and this may have several consequences:

- Boundaries between objects can be imprecisely defined, leading to handling fuzzy objects instead of crisp ones. Therefore, the measures that will be defined in this paper need to apply for both crisp and fuzzy objects.
- The objects may be only partially detected, which limits drastically measures based only on size and shape of the objects. This limit is even more severe for objects having similar shapes.

In such cases, individual recognition of each object is almost impossible, or would be unreliable. Therefore, the features used in the recognition process have to make use of the structural information, i.e. the spatial arrangement of the objects in the scene. Indeed, spatial relationships may allow to recognize objects with reference to other ones, even if individual properties of the objects are not reliable.

We distinguish two kinds of spatial relationships. Some of them are well defined if the objects are crisp, like adjacency, inclusion or other set relationships. But since they are highly sensitive to errors or imprecision in segmentation, more useful measures can be obtained by fuzzifying these concepts.

Other relationships are inherently vague concepts, like relative position, surroundness, betweenness, etc. Fuzzy definitions of such relationships are then more consistent than crisp ones.

In both cases, fuzzy sets appear as an appropriate tool for providing consistent definitions that fit the intuition and have good properties.

III. Extending well defined crisp relationships to fuzzy sets

In this Section, we propose fuzzy definitions generalizing well defined crisp spatial relationships. Such crisp concepts have a limited interest in case of imprecise segmentation, since they are very sensitive to tiny changes in object delineation. For instance, relationships like adjacency or inclusion may be satisfied between two objects, and no more satisfied if only a few points are missing or added in one of the objects. This can be avoided by dealing with fuzzy objects, taking explicitly into account imprecision in segmentation, and by generalizing classical definitions to such objects.

A. General principle

In this Section, we consider the general problem of extending a relationship R_B between two binary objects to its fuzzy equivalent R (fuzzy relationship between two fuzzy objects).

One way to define crisp sets from a fuzzy set consists in taking the α-cuts of this set. Therefore a first class of methods relies on the application of the relationship R_B to each α-cut. This gives rise to two different "fuzzification" methods in the literature.

The first one consists in "stacking" the results obtained with binary operations on the α-cuts: let us denote by μ and ν the membership functions of two fuzzy objects defined on the considered space and taking values in [0,1], the fuzzy equivalent R of R_B is then defined as (see e.g. [5], [7], [12]):

$$R(\mu, \nu) = \int_0^1 R_B(\mu_\alpha, \nu_\alpha) d\alpha, \quad (1)$$

or similarly by a double integration. Other fuzzification equations are also possible. Examples of this approach concern for instance connectivity [14], fuzzy mathematical morphology [5], distances [7], [1], [4], etc.

The second method is derived from the extension principle [17], which leads in the general case to a fuzzy number (rather than a crisp number):

$$\forall n \in \mathcal{V}(R_B), R(\mu, \nu)(n) = \sup_{R_B(\mu_\alpha, \nu_\alpha) = n} \alpha, \quad (2)$$

where $\mathcal{V}(R_B)$ denotes the image of R_B, i.e. the set of values taken by R_B. If the relationship to be extended only takes binary values (0/1, or true/false), then this expression reduces to a number.

A second class of methods consists in translating binary equations into their fuzzy equivalent: intersection is replaced by a t-norm, union by a t-conorm, sets by membership functions, etc. Examples can be found for defining fuzzy morphology [5], fuzzy inclusion [16], etc.

These two classes of methods may lead to the same definitions for particular choices of the fuzzy operators.

B. Using fuzzy mathematical morphology

The extension method using formal translation of equations is particularly straightforward if the binary relationship can be expressed in set theoretical and logical terms. This is actually the case for set relationships and for several others, like adjacency. Moreover, this remark endows methods based on mathematical morphology with a particular interest, since mathematical morphology is mainly based on set theory. Therefore all relationships that can be expressed in terms of morphological operators (Hausdorff distance for instance) will be easily extended to fuzzy objects.

In the following, we define fuzzy relationships by extending the corresponding crisp ones using formal translation of set theoretical and morphological expressions.

C. Adjacency between fuzzy sets

Adjacency is an example of spatial relationship that is well defined in the crisp case and that carries a strong information about the structure of the image or the scene.

In the crisp discrete case, two image regions X and Y are adjacent if:

$$X \cap Y = \emptyset \textit{ and } \exists x \in X, \exists y \in Y : n_c(x, y), \quad (3)$$

where $n_c(x, y)$ is the Boolean variable stating that x and y are neighbors in the sense of the discrete c-connectivity. A consequence of this definition is that, if X and Y are adjacent, then any $x \in X$ and $y \in Y$ that satisfy $n_c(x, y)$ belong to the boundary of X and Y respectively. Definition 3 can also be expressed equivalently in terms of morphological dilation, as: $X \cap Y = \emptyset$ *and* $D(X, B_c) \cap Y \neq \emptyset$, $D(Y, B_c) \cap X \neq \emptyset$, where $D(X, B_c)$ denotes the dilation of X by the elementary structuring element B_c, corresponding to the c-connectivity.

The extension of this definition can therefore be performed in three different ways, as detailed in [6]. This extension involves the definitions of a degree of intersection $\mu_{int}(\mu, \nu)$ between two fuzzy sets μ and ν defined on $\mathcal{S}$, as well as a degree of non-intersection $\mu_{\neg int}(\mu, \nu)$, and a degree of neighborhood n_{xy} between two points x and y of $\mathcal{S}$. This leads to the following definition for fuzzy adjacency between μ and ν:

$$\mu_{adj}(\mu, \nu) = t[\mu_{\neg int}(\mu, \nu), \sup_{x \in \mathcal{S}} \sup_{y \in \mathcal{S}} t[\mu(x), \nu(y), n_{xy}]]. \quad (4)$$

Adding the constraint on boundary leads to the following definition:

$$\mu_{adj}(\mu, \nu) = t[\mu_{\neg int}(\mu, \nu), \sup_{x \in \mathcal{S}} \sup_{y \in \mathcal{S}} t[b_\mu(x), b_\nu(y), n_{xy}]], \quad (5)$$

where b_μ denotes the fuzzy boundary of μ (see [6] for details).

Finally, the degree of adjacency between μ and ν involving fuzzy dilation is defined as:

$$\mu_{adj}(\mu, \nu) = t[\mu_{\neg int}(\mu, \nu), \mu_{int}[D(\mu, B_c), \nu], \mu_{int}[D(\nu, B_c), \mu]], \quad (6)$$

where $D(\mu, B_c)$ denotes the fuzzy dilation of μ by the structuring element B_c [5]. This definition represents a conjunctive combination of a degree of non-intersection between μ and ν and a degree of intersection between one fuzzy set and the dilation of the other. B_c can be taken as the elementary structuring element related to the considered connectivity, or as a fuzzy structuring element, representing for instance spatial imprecision (i.e. the possibility distribution of the location of each point).

All these definitions have the desirable properties: they are symmetrical, consistent with the binary definition if μ, ν and B_c are binary, and decreases if the distance between μ and ν increases. There are also invariant with respect to geometrical transformations.

D. Distances between fuzzy sets

Distance between fuzzy sets is an example where several mathematical definitions exist in the binary case. Here we show that the construction principle based on translation of formal expressions can be applied easily as soon as the distance can be expressed in set theoretical and logical terms. It is the case for distances having a direct expression in terms of mathematical morphology (e.g. nearest point distance, Hausdorff distance, or distance from a point to a set). Therefore we defined in [1] the fuzzy equivalent of these distances in terms of fuzzy mathematical morphology.

We consider here the examples of nearest point distance (denoted by d_N) and Hausdorff distance (denoted by d_H) since they have a direct morphological expression that will be used in the following:

$$\begin{aligned} d_N(X,Y) &= \inf\{n, X \cap D^n(Y) \neq \emptyset\} \\ &= \inf\{n, Y \cap D^n(X) \neq \emptyset\}. \end{aligned} \quad (7)$$

$$d_H(X,Y) = \inf\{n, X \subset D^n(Y) \text{ and } Y \subset D^n(X)\}. \quad (8)$$

In these equations, X and Y denote two crisp sets of the considered space $\mathcal{S}$, and $D^n(X)$ the dilation of size n of X. Note that for the nearest point distance, separability and triangular inequality are not satisfied, while the Hausdorff distance is a true distance.

A direct morphological expression of the fuzzy extension of nearest point distance is obtained by translating equation 7. We define a distance distribution $\Delta_N(\mu,\mu')(n)$ that expresses the degree to which the distance between two fuzzy sets μ and μ' is less than n by:

$$\begin{aligned} \Delta_N(\mu,\mu')(n) = f[&\sup_{x\in\mathcal{S}} t[\mu(x), D_\nu^n(\mu')(x)], \\ &\sup_{x\in\mathcal{S}} t[\mu'(x), D_\nu^n(\mu)(x)]], \end{aligned} \quad (9)$$

where t is a t-norm, f is a symmetrical function, and D_ν^n denotes the fuzzy dilation of size n by the fuzzy structuring element ν. A distance density, i.e. a fuzzy number $\delta_N(\mu,\mu')(n)$ representing the degree to which the distance between μ and μ' is equal to n can be obtained implicitly by [15]:

$$\Delta_N(\mu,\mu')(n) = \int_0^n \delta_N(\mu,\mu')(n')dn'. \quad (10)$$

Clearly, this expression is not very tractable and does not lead to a simple explicit expression of $\delta_N(\mu,\mu')(n)$. Therefore, we suggest to use an explicit method, exploiting the fact that, for $n > 0$, we have:

$$d_N(X,Y) = n \Leftrightarrow D^n(X) \cap Y \neq \emptyset \text{ and } D^{n-1}(X) \cap Y = \emptyset \quad (11)$$

and the symmetrical expression. For $n = 0$ we have:

$$d_N(X,Y) = 0 \Leftrightarrow X \cap Y \neq \emptyset. \quad (12)$$

The translation of these equivalences provides, for $n > 0$:

$$\begin{aligned} \delta_N(\mu,\mu')(n) = t[&\sup_{x\in\mathcal{S}} t[\mu'(x), D_\nu^n(\mu)(x)], \\ &c[\sup_{x\in\mathcal{S}} t[\mu'(x), D_\nu^{n-1}(\mu)(x)]]] \end{aligned} \quad (13)$$

or a symmetrical expression derived from this one, and:

$$\delta_N(\mu,\mu')(0) = \sup_{x\in\mathcal{S}} t[\mu(x), \mu'(x)]. \quad (14)$$

From equation 8, a distance distribution for Hausdorff distance can be defined, by introducing fuzzy dilation:

$$\begin{aligned} \Delta_H(\mu,\mu')(n) = t[&\inf_{x\in\mathcal{S}} T[D_\nu^n(\mu)(x), c(\mu'(x))], \\ &\inf_{x\in\mathcal{S}} T[D_\nu^n(\mu')(x), c(\mu(x))]], \end{aligned} \quad (15)$$

where c is a complementation, t a t-norm and T the dual t-conorm. Like for the nearest point distance, a distance density can be derived implicitly from this distance distribution. A direct definition of a distance density can be obtained from:

$$d_H(X,Y) = 0 \Leftrightarrow X = Y, \quad (16)$$

and for $n > 0$:

$$\begin{aligned} d_H(X,Y) = n \Leftrightarrow & X \subset D^n(Y) \text{ and } Y \subset D^n(X) \\ & \text{and } \left(X \not\subset D^{n-1}(Y) \text{ or } Y \not\subset D^{n-1}(X)\right). \end{aligned} \quad (17)$$

Translating these equations leads to a definition of the Hausdorff distance between two fuzzy sets μ and μ' as a fuzzy number:

$$\delta_H(\mu,\mu')(0) = t[\inf_{x\in\mathcal{S}} T[\mu(x), c(\mu'(x))], \inf_{x\in\mathcal{S}} T[\mu'(x), c(\mu(x))]], \quad (18)$$

$$\begin{aligned} \delta_H(\mu,\mu')(n) = t[&\inf_{x\in\mathcal{S}} T[D_\nu^n(\mu)(x), c(\mu'(x))], \\ &\inf_{x\in\mathcal{S}} T[D_\nu^n(\mu')(x), c(\mu(x))], \\ T(&\sup_{x\in\mathcal{S}} t[\mu(x), D_\nu^{n-1}(\mu')(x)], \sup_{x\in\mathcal{S}} t[\mu'(x), D_\nu^{n-1}(\mu)(x)])]. \end{aligned} \quad (19)$$

The previous definitions of fuzzy nearest point and Hausdorff distances (defined as fuzzy numbers) between two fuzzy sets do not necessarily share the same properties as their crisp equivalent. This is due in particular to the fact that, depending on the choice of the involved t-norms and t-conorms, excluded middle and non contradiction laws may not be satisfied. All distances are positive, in the sense that the defined fuzzy numbers have always a support included in the positive half real line. By construction, all defined distances are symmetrical with respect to μ and μ'. The separability property is not always satisfied. However, if μ is normalized, we have for the nearest point distance $\delta_N(\mu,\mu)(0) = 1$ and $\delta_N(\mu,\mu)(n) = 0$ for $n > 1$. For the Hausdorff distance, $\delta_H(\mu,\mu')(0) = 1$ implies $\mu = \mu'$ for T being the bounded sum ($T(a,b) = \min(1, a+b)$), while it implies μ and μ' crisp and equal for $T = \max$. Also the triangular inequality is not satisfied in general.

IV. Fuzzy definition for intuitive but mathematically vague relationships: relative position

In this Section, we consider relative position, as a good example for the second type of relationships. Indeed, relationships like "left to" are rather ambiguous concepts. They defy precise definitions, but human beings have a rather intuitive and common way of understanding and interpreting them, and they are of prime importance for understanding the structure of a scene. From our every day experience, it is clear that any "all-or-nothing" definition leads to unsatisfactory results in several situations, even of moderate complexity. Therefore, relative position concepts may find a better understanding in the framework of fuzzy sets, as fuzzy relationships. Several definitions have been proposed in the literature, most of them relying on histograms of angles between objects [12], [13], [10]. Here, we present an original definition based on a morphological approach, and dealing with crisp as well as fuzzy objects, and that works in 2D as well as 3D.

Let us consider a reference object R and an object A for which the relative position with respect to R has to be evaluated. In order to evaluate the degree to which A is in some direction with respect to R, we propose the following approach:

1. We first define a fuzzy "landscape" around the reference object R as a fuzzy set such that the membership value of each point corresponds to the degree of satisfaction of the spatial relation under examination. The fuzzy landscape is directly defined in the same space as the considered objects.
2. We then compare the object A to the fuzzy landscape attached to R, in order to evaluate how well the object matches with the areas having high membership values (i.e. areas that are in the desired direction). This is done using a fuzzy pattern matching approach, which provides an evaluation as an interval instead of one number only. This makes a major difference with respect to all the previous approaches, and, to our opinion, it provides a richer information about the considered relationship.

In the 3D Euclidean space, a direction is defined by two angles α_1 and α_2, where $\alpha_1 \in [0, 2\pi]$ and $\alpha_2 \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ ($\alpha_2 = 0$ in the 2D case). The direction in which the relative position of an object with respect to another one is evaluated is denoted by: $\vec{u}_{\alpha_1,\alpha_2} = (\cos\alpha_2 \cos\alpha_1, \cos\alpha_2 \sin\alpha_1, \sin\alpha_2)^t$, and we note $\alpha = (\alpha_1, \alpha_2)$.

We consider two (possibly fuzzy) objects, R and A, and define the degree to which A is in direction $\vec{u}_{\alpha_1,\alpha_2}$ with respect to R. Let us denote by $\mu_\alpha(R)$ the fuzzy set defined in the image in such a way that points of areas which satisfy to a high degree the relation "to be in the direction $\vec{u}_{\alpha_1,\alpha_2}$ with respect to reference object R" have high membership values. In other terms, the membership function $\mu_\alpha(R)$ has to be an increasing function of the degree of satisfaction of the relation. It is a spatial fuzzy set (i.e. a function of the image $\mathcal{S}$ into [0,1]) and directly related to the shape of R. The precise definition of $\mu_\alpha(R)$ is given later.

Let us denote by μ_A the membership function of the object A, which is a function of $\mathcal{S}$ into [0,1]. The evaluation of relative position of A with respect to R is given by a function of $\mu_\alpha(R)(x)$ and $\mu_A(x)$ for all $x \in \mathcal{S}$. An appropriate tool for defining this function is the fuzzy pattern matching approach [9]. Following this approach, the evaluation of the matching between two possibility distributions consists of two numbers, a necessity degree N (a pessimistic evaluation) and a possibility degree Π (an optimistic evaluation), as often used in the fuzzy set community. In our application, they take the following forms:

$$\Pi^R_{\alpha_1,\alpha_2}(A) = \sup_{x\in\mathcal{S}} t[\mu_\alpha(R)(x), \mu_A(x)],$$

$$N^R_{\alpha_1,\alpha_2}(A) = \inf_{x\in\mathcal{S}} T[\mu_\alpha(R)(x), 1-\mu_A(x)], \qquad (20)$$

where t is a t-norm (fuzzy intersection) and T a t-conorm (fuzzy union) [8]. In the crisp case, these equations reduce to: $\Pi^R_{\alpha_1,\alpha_2}(A) = \sup_{x\in A} \mu_\alpha(R)(x)$, and $N^R_{\alpha_1,\alpha_2}(A) = \inf_{x\in A} \mu_\alpha(R)(x)$.

The possibility corresponds to a degree of intersection between the fuzzy sets A and $\mu_\alpha(R)$, while the necessity corresponds to a degree of inclusion of A in $\mu_\alpha(R)$. They can also be interpreted in terms of fuzzy mathematical morphology, since the possibility $\Pi^R_{\alpha_1,\alpha_2}(A)$ is equal to the dilation of μ_A by $\mu_\alpha(R)$ at origin, while the necessity $N^R_{\alpha_1,\alpha_2}(A)$ is equal to the erosion, as shown in [5]. These two interpretations, in terms of set theoretic operations and in terms of morphological ones, explain how the shape of the objects is taken into account.

Several other functions combining $\mu_\alpha(R)$ and $\mu_A(x)$ can be constructed. The extreme values provided by the fuzzy pattern matching are interesting because of their morphological interpretation, and because they provide an interval and not only a single value and may represent in this way the ambiguity of the relation if any. An average measure can also be useful from a practical point of view, and is defined as:

$$M^R_{\alpha_1,\alpha_2}(A) = \frac{1}{|A|}\sum_{x\in\mathcal{S}} \mu_A(x)\mu_\alpha(R)(x), \qquad (21)$$

where $|A|$ denotes the fuzzy cardinality of A: $|A| = \sum_{x\in\mathcal{S}} \mu_A(x)$.

The key point in the previous definition relies in the definition of $\mu_\alpha(R)$. The requirements for this fuzzy set are not strong and leave room for a large spectrum of possibilities. This flexibility allows the user to define any membership function according to the application at hand and the context requirements. We propose here a definition that looks precisely at the domains of space that are visible from a reference object point in the direction $\vec{u}_{\alpha_1,\alpha_2}$. This applies to any kind of objects, in particular having strong concavities. Let us denote by P any point of $\mathcal{S}$, and by Q any point of R. Let $\beta(P,Q)$ be the angle between the vector $\vec{QP}$ and the direction $\vec{u}_{\alpha_1,\alpha_2}$, computed in $[0,\pi]$. We then determine for each point P the point Q of R leading to the smallest angle β, denoted by $\beta_{\min}$. In the crisp case, this point Q is the reference object point from which P is visible in the direction the closest to $\vec{u}_{\alpha_1,\alpha_2}$: $\beta_{\min}(P) = \min_{Q\in R}\beta(P,Q)$. The fuzzy landscape $\mu_\alpha(R)$ at point P is then defined as: $\mu_\alpha(R)(P) = f(\beta_{\min}(P))$, where f is a decreasing function of $[0,\pi]$ into $[0,1]$. In our experiments, we have chosen a simple linear function: $\mu_\alpha(R)(P) = \max(0, 1 - \frac{2\beta_{\min}(P)}{\pi})$.

In the fuzzy case, we propose a method which only combines membership values, one describing the membership to R, and the other to the fuzzy landscape. This corresponds to translating binary equations and propositions into fuzzy ones: in the binary case, we express that: $Q \in R$ and $f(\beta_{\min}) = \max_{Q\in R} f(\beta(P,Q))$ (since f is decreasing), which translates in fuzzy terms as:

$$\mu_\alpha(R)(P) = \max_{Q\in Supp(\mathcal{S})} t[\mu_R(Q), f(\beta(P,Q))], \tag{22}$$

where t is a t-norm.

An advantage of this approach is its interpretation in terms of morphological operations. It can be shown that $\mu_\alpha(R)$ is exactly the fuzzy dilation of μ_R by ν, where ν is a fuzzy structuring element defined on $\mathcal{S}$ as:

$$\forall P \in \mathcal{S},\ \nu(P) = \max[0, 1 - \frac{2}{\pi}\arccos\frac{\vec{OP}\cdot\vec{u}_\alpha}{||\vec{OP}||}], \tag{23}$$

where O is the center of the structuring element. This equivalence provides an additional morphological interpretation of our definition. The reader may refer to [3] for more details on these definitions and their properties.

V. Application in brain imaging

In this Section, we illustrate the method on a fuzzy example taken from medical imaging, which shows more practical properties of the proposed approach. In a magnetic resonance (MR) image of the human brain we have segmented several internal structures using a fuzzy segmentation method. Five fuzzy structures are shown in Figure 1 (with the standard "left-is-right" convention of medical images): left ventricle (v1), right ventricle (v2), left caudate nucleus (nc1), right caudate nucleus (v2) and left thalamus (t1). These structures may be recognized using an anatomical atlas for instance, by comparing relationships between atlas structures and relationships between image structures.

The adjacency degrees between some of the obtained fuzzy objects are given in Table I. The results are in agreement with what can be expected from the model (crisp adjacency between atlas objects). In this case, crisp adjacency would provide completely different results in the model and in the image, preventing its use for recognition. This suggests that fuzzy adjacency degree can indeed be used for pattern recognition purposes, of course combined with other spatial relationships.

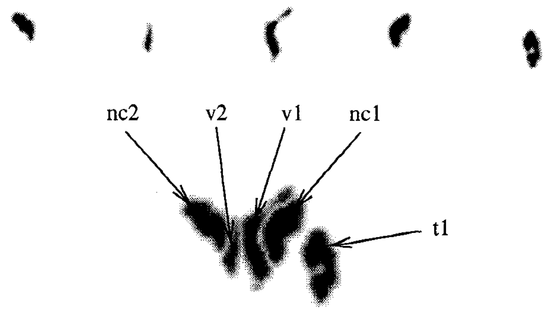


Fig. 1. Top: 5 fuzzy objects resulting from a rough fuzzy segmentation of a MR brain image (membership values rank between 0 and 1, from white to black). Bottom: superposition of these fuzzy objects and labels as observed in the original MR image.

Fuzzy object 1	Fuzzy object 2	degree of adjacency	adjacency in the model (crisp)
v1	v2	0.368	1
v1	nc1	0.463	1
v1	t1	0.000	0
v1	nc2	0.035	0
v2	nc2	0.427	1
nc1	t1	0.035	0

TABLE I

Results obtained for fuzzy adjacency. Labels of structures are given in Figure 1. High degrees are obtained between structures where adjacency is expected, while very low degrees are obtained in the opposite case.

Examples of distances between brain structures are shown in Table II, as fuzzy numbers issued from the morphological definitions. The results are in agreement with what is expected: v2 is near from nc2 and v1, quite far from nc1 and very far from t1. We do not obtain a null value for v2, since it does not perfectly match the model of v2, but we obtain values that are still much lower than those obtained for the other structures. This shows that distances can be used both for identifying a structure among several possible ones, by using distance as a dissimilarity measure, and for describing the spatial arrangement of objects in terms of nearness or farness.

The fuzzy landscapes representing the degree of satisfaction of the relations "left to", "right to", " below" and "above" object v1 are shown in Figure 2. They are obtained using Equation 22 using the product t-norm. The relative position degrees between some of the obtained fuzzy objects are given in Figure 3, for the t-norm min in the fuzzy pattern matching. The interpretation of these results is straightforward with respect to the intuitive expected relative positions. Object nc1 is mainly to the right

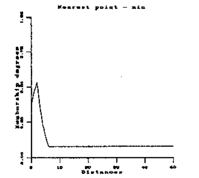
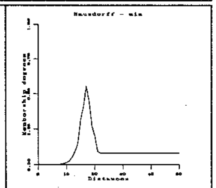
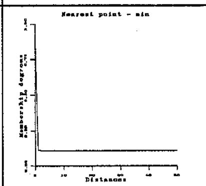
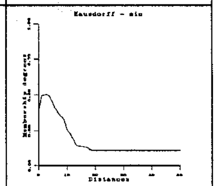
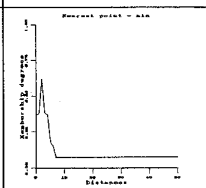
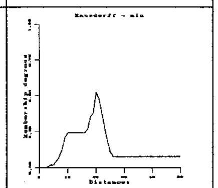
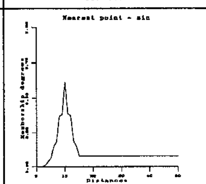
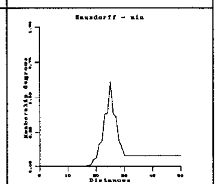
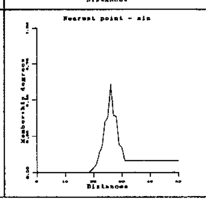
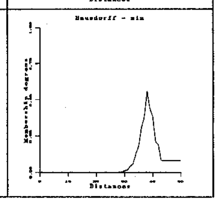
Distance between v2 and:		
object	δ_N (min)	δ_H (min)
nc2		
v2		
v1		
nc1		
t1		

TABLE II

DISTANCES BETWEEN FUZZY SETS USING THE MORPHOLOGICAL APPROACH, FOR THE NEAREST POINT DISTANCE AND THE HAUSDORFF DISTANCE, USING MINIMUM AS T-NORM. THE DISTANCE IS COMPUTED BETWEEN EACH OF THE 5 STRUCTURES AND A MODEL OF V2.

of v1 (and only with very low degree to its left), and quite above and below. This expresses that it is "in the right concavity of v1", an example of more complex relationship derived from the basic relative positions. Object nc2 is to the left of v1, with no ambiguity at all concerning the right relationship (i.e. no point of nc2 is to the right of v1). It is quite above v1, and less below it than nc1. Similar interpretations can be given for t1 and v2 with respect to v1.

VI. Conclusion

We have shown is this paper the power of fuzzy set theory for describing structural information in images in the form of spatial relationships between imprecise image objects. Further work aims at integrating these relationships in a structural model-based pattern recognition system, for

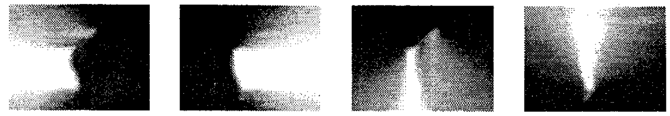


Fig. 2. Fuzzy areas corresponding to four relationships of Figure 3 for the object v1 of Figure 1.

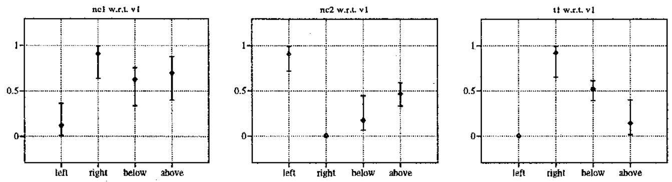


Fig. 3. Results of relative position obtained for some of the objects of Figure 1.

spatial reasoning under imprecision.

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