Fuzzy Spatial Relationships for Model-Based Pattern Recognition in Images and Spatial Reasoning Under Imprecision

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Abstract. We show in this paper that mathematical morphology provides a unified and consistent framework to express different types of spatial relationships and to answer different questions about them, with good properties. We show then how to use these fuzzy relationships in model-based pattern recognition and spatial reasoning under imprecision. Two examples are presented, one where recognition of face features is expressed as non bijective correspondence between graphs representing regions and spatial relations, and one where anatomical expert knowledge involving spatial relationships is used to guide the recognition of brain structures.

1 Introduction

Fuzzy set theory provides a good theoretical basis to represent imprecision of information, at different levels of representation, in particular in image processing and interpretation. It constitutes a unified framework for representing and processing both numerical and symbolic information, as well as structural information (constituted mainly by spatial relationships in image processing). The interest of spatial relationships between objects has been highlighted in very different types of works. Indeed, the spatial arrangement of objects in images provides important information for recognition and interpretation tasks, in particular when the objects are embedded in a complex environment like in medical or remote sensing images. We distinguish between relationships that are mathematically well defined and relationships that are intrinsically vague. Topological relationships (such as set relationships and adjacency) and distances belong to the first class. If the objects are precisely defined, their relationships can be defined and computed in a numerical (purely quantitative) setting. But if the objects are imprecise, as is often the case if they are extracted from images, then the semi-quantitative framework of fuzzy sets proved to be useful for their representation, as spatial fuzzy sets. Definitions of relationships have then to be extended to be applicable on fuzzy objects. Results can also be semi-quantitative, and provided in the form of intervals or fuzzy numbers. Some metric relationships, like relative directional position, belong to the second class. Even for crisp objects, fuzzy definitions are then appropriate.

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We show in this paper that mathematical morphology provides a unified and consistent framework to express different types of spatial relationships and to answer different questions about them, with good properties (Section 2). We show then through two examples how to use these fuzzy relationships in model-based pattern recognition and spatial reasoning under imprecision (Section 3).

2 Mathematical Morphology as a Unified Framework for Defining Spatial Relationships

In this Section we address the problem of modeling spatial relationships in the fuzzy set framework. This framework is interesting here for several reasons:

- the objects of interest can be imprecisely defined, for instance due to the segmentation step;
- some relations are imprecise, such as *to be left of*, and find a more suitable definition in the fuzzy set framework;
- the type of knowledge available about the structures or the type of question we would like to answer can be imprecise too.

We consider here adjacency (as one example of topological relation), distances, and directional relative position. Some of them have led to a rich literature in the fuzzy set community, like distances which have been defined using a lot of different approaches, while others have not raised so much attention. We summarize here our work based on fuzzy mathematical morphology [1], which allows us to represent in a unified way various spatial relationships [2].

Two types of questions are important for applications in structural pattern recognition:

1. given two objects (possibly fuzzy), assess the degree to which a relation is satisfied;
2. given one reference object, define the area of the space in which a relation to this reference is satisfied (to some degree).

Our approach provides answers to these two types of questions. The second one will be illustrated only for distances and directional position here (see [2] for the other relations).

We consider the general case of a 3D space $\mathcal{S}$ (typically $\mathbb{R}^3$ or $\mathbb{Z}^3$ in the digital case), where objects can have any shape and any topology, and can be crisp or fuzzy.

Adjacency. Adjacency has a large interest in image processing and pattern recognition, since it denotes an important relationship between image objects or regions [3], widely used as a feature in model-based pattern recognition. In the crisp case, it is defined based on the digital connectivity $n_c(x, y)$ defined on the image: two subsets X and Y in $\mathcal{S}$ are adjacent according to the c-connectivity if: $X \cap Y = \emptyset$ *and* $\exists x \in X, \exists y \in Y : n_c(x, y)$. This definition can also be expressed equivalently in terms of morphological dilation, as: $X \cap Y = \emptyset$ *and* $D_B(X) \cap Y \neq$

$\emptyset$, $D_B(Y) \cap X \neq \emptyset$, where B denotes the elementary structuring element associated to the c-connectivity and $D_B(X)$ denotes the dilation of X by B.

This morphological expression can be extended to the fuzzy case, leading to the following degree of adjacency between two fuzzy sets [4]:

$$\mu_{adj}(\mu, \nu) = t[\mu_{\neg int}(\mu, \nu), \mu_{int}[D_B(\mu), \nu], \mu_{int}[D_B(\nu), \mu]], \quad (1)$$

where $D_B(\mu)$ denotes the fuzzy dilation of μ by B. This definition represents a conjunctive combination (through a t-norm t) of a degree of non-intersection $\mu_{\neg int}$ between μ and ν and a degree of intersection μ_{int} between one fuzzy set and the dilation of the other. B can be taken as the elementary structuring element related to the considered connectivity, or as a fuzzy structuring element, representing for instance spatial imprecision (i.e. the possibility distribution of the location of each point). We proved that this definition is symmetrical, consistent with the binary definition if μ, ν and B are binary, decreases if the distance between μ and ν increases, and is invariant with respect to geometrical transformations [4].

Distances. The importance of distances in image processing is well established. Their extensions to fuzzy sets can be useful in several parts of image processing under imprecision (classification and clustering, skeletonization, registration, structural pattern recognition, since distances constitute a major component of the spatial arrangement of objects).

Several definitions can be found in the literature for distances between fuzzy sets (which is the main addressed problem). They can be roughly divided in two classes: distances that take only membership functions into account and that compare them point-wise, and distances that additionally include spatial distances [5]. The definitions which combine spatial distance and fuzzy membership comparison allow for a more general analysis of structures in images, for applications where topological and spatial arrangement of the structures of interest is important (segmentation, classification, scene interpretation). These distances combine membership values at different points in the space, and take into account their proximity or distance in $\mathcal{S}$. The price to pay is an increased complexity, generally quadratic in the cardinality of $\mathcal{S}$.

We proposed in [5] original approaches for defining fuzzy distances taking into account spatial information, which are based on fuzzy mathematical morphology. The idea is that in the binary case, there exist strong links between mathematical morphology (in particular dilation) and distances (from a point to a set, and between two sets), and this can also be exploited in the fuzzy case. The advantage is that distances are expressed in set theoretical terms, and are therefore easier to translate to the fuzzy case with nice properties than usual analytical expressions. Definitions of nearest point distance between two fuzzy sets and of Hausdorff distance can be obtained this way, and provide evaluations as fuzzy numbers. These definitions are not detailed here, but the reader can refer to [5] for these, as well as for properties and examples.

Let us now consider the second question, i.e. defining the area of space that satisfies some distance property with respect to a reference object. We assume that a set A is known as one already recognized object, or a known area of $\mathcal{S}$, and we want to determine B, subject to satisfy some distance relationship with A. According to the algebraic expressions of distances, dilation of A is an adequate tool for this. For instance if the knowledge expresses that $d(A,B) \geq n$, then B should be looked for in $D^{n-1}(A)^C$ ($D^n(A)$ denotes the dilation of size n of A). As another example, expressing that B should lay between a distance n_1 and a distance n_2 of A can be obtained by considering both minimum and maximum (Hausdorff) distances: the minimum distance should be greater than n_1 and the maximum distance should be less than n_2. In this case, the volume of interest for B is reduced to $D^{n_2}(A) \setminus D^{n_1-1}(A)$.

In cases where imprecision has to be taken into account, fuzzy dilations are used, with the corresponding equivalences with fuzzy distances [1, 5]. The extension to approximate distances calls for fuzzy structuring elements. We define these structuring elements through their membership function ν on $\mathcal{S}$. Structuring elements with a spherical symmetry can typically be used, where the membership degree only depends on the distance to the center of the structuring element.

Let us consider the generalization to the fuzzy case of the last case (minimum distance of at least n_1 and maximum distance of at most n_2 to a fuzzy set μ). Instead of defining an interval $[n_1, n_2]$, we consider a fuzzy interval, defined as a fuzzy set on $\mathbb{R}^+$ having a core equal to the interval $[n_1, n_2]$. The membership function μ_n is increasing between 0 and n_1 and decreasing after n_2 (this is but one example). Then we define two structuring elements, as:

$$\nu_1(v) = \begin{cases} 1 - \mu_n(d_E(v,0)) & \text{if } d_E(v,0) \leq n_1 \\ 0 & \text{else} \end{cases} \tag{2}$$

$$\nu_2(v) = \begin{cases} 1 & \text{if } d_E(v,0) \leq n_2 \\ \mu_n(d_E(v,0)) & \text{else} \end{cases} \tag{3}$$

where d_E is the Euclidean distance in $\mathcal{S}$ and O the origin. The spatial fuzzy set expressing the approximate relationship about distance to μ is then defined as:

$$\mu_{distance} = t[D_{\nu_2}(\mu), 1 - D_{\nu_1}(\mu)] \tag{4}$$

if $n_1 \neq 0$, and $\mu_{distance} = D_{\nu_2}(\mu)$ if $n_1 = 0$. The increasingness of fuzzy dilation with respect to both the set to be dilated and the structuring element [1] guarantees that these expressions do not lead to inconsistencies: we have $\nu_1 \subseteq \nu_2$, $\nu_1(0) = \nu_2(0) = 1$, and therefore $\mu \subseteq D_{\nu_1}(\mu) \subseteq D_{\nu_2}(\mu)$. In the case where $n_1 = 0$, we do not have $\nu_1(0) = 1$ any longer, but in this case, only the dilation by ν_2 is considered. This case corresponds actually to a distance to μ less than "about n_2". These properties are indeed expected for representations of distance knowledge.

Directional Relative Position. This type of relation is ambiguous and imprecise even if objects are crisp. Therefore, relative position concepts may find a better understanding in the framework of fuzzy sets, as fuzzy relationships, even for crisp objects. This framework makes it possible to propose flexible definitions which fit the intuition and may include subjective aspects, depending on the application and on the requirements of the user. The few existing fuzzy approaches in the literature mostly rely on angle histogram [6, 7] or extensions of it [8]. Our approach is completely different since it works directly in the spatial domain.

Let us consider a reference object R and an object A for which the relative position with respect to R has to be evaluated. In order to evaluate the degree to which A is in some direction with respect to R, we propose the following approach [9]:

1. We first define a fuzzy "landscape" around the reference object R as a fuzzy set such that the membership value of each point corresponds to the degree of satisfaction of the spatial relation under examination. This is formally defined by a fuzzy dilation of R by a fuzzy structuring element representing the desired relation with respect to the origin.
2. We then compare the object A to the fuzzy landscape attached to R, in order to evaluate how well the object matches with the areas having high membership values (i.e. areas that are in the desired direction). This is done using a fuzzy pattern matching approach [10], providing an evaluation consisting of two numbers, a necessity degree N (a pessimistic evaluation) and a possibility degree Π (an optimistic evaluation), as often used in the fuzzy set community. An average measure can also be useful from a practical point of view.

The first step answers the second type of question, while the second one answers the first type. Details about the formalization and the properties, as well as some algorithmical and computational aspects, can be found in [9].

3 Examples in Model-Based Pattern Recognition and Spatial Reasoning

Spatial reasoning is a research field dedicated to reasoning about spatial entities and spatial relations. It is particularly developed in artificial intelligence and several formal theories have been developed (see e.g. [11] for a survey) but much less in image interpretation. An example of application concerns structural recognition in images under imprecision.

Let us now briefly illustrate how fuzzy spatial relations can be used for recognizing structures in a scene based on a model of this scene. Two types of approaches can be developed, corresponding to the two types of questions mentioned in Section 2.

Graph-Based Approach. In the first approach, spatial relations evaluated between spatial entities (typically objects or regions) are considered as attributes in a graph.

Graph representations are widely used for dealing with structural information, in different domains including image interpretation and model-based pattern recognition. Here, we assume that the model is represented as a graph where nodes are objects and edges represent links between these objects. Both nodes and edges are attributed. Node attributes are characteristics of the objects, while edge attributes quantify spatial relationships between the objects. A data graph is then constructed from each image where the recognition has to be performed. Since it is usually difficult to segment directly the objects, usually the graph is based on an over-segmentation of the image, for instance based on watersheds. Each region constitutes a node of this data graph, and edges represent links between regions. Attributes are computed as for the model. The use of fuzzy relations is particularly useful in order to be less sensitive to the segmentation.

One important problem to be solved then is graph matching. In order to achieve a good correspondence between both graphs, the most used concept is the one of graph isomorphism and a lot of work is dedicated to the search for the best isomorphism between two graphs or subgraphs. However, in a number of cases, the bijective condition is too strong: because of the schematic aspect of the model and of the difficulty to segment the image into meaningful entities, no isomorphism can be expected between both graphs. In particular, several regions of the image can be assigned to the same node of the model graph. Such problems call for inexact graph matching. It constitutes generally in finding a morphism, which furthermore optimizes an objective function based on similarities between attributes. The morphism aims at preserving the structure of the graphs, while the objective function privileges the association between nodes, respectively between edges, with similar attribute values. This approach can benefit from the huge literature on fuzzy comparison tools (see e.g. [12]) and from recent developments on fuzzy morphisms [13]. The optimization is not an easy task since the problem is NP-hard. Genetic algorithms, estimation of distribution algorithms (EDA) and tree search methods have been developed towards this aim [14, 15, 16].

This approach has been applied in brain imaging, in order to recognize brain structures in a 3D magnetic resonance image (MRI) based on an anatomical atlas [14], and in face feature recognition, based on a rough model of a face constructed from a different person image [16] (an example is shown in Figure 1).

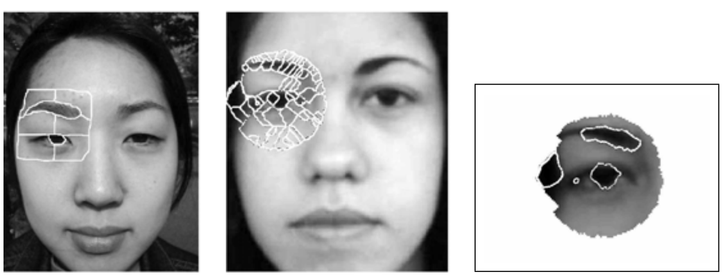

Fig. 1. Left: model; middle: over-segmented image (subset); right: results on a few face features obtained with EDA (from [16])

Focusing Attention Based on Spatial Representation of Spatial Knowledge. In the second type of approach, we use the spatial representation of spatial knowledge. Each relation is then represented as a spatial fuzzy set, constraining the search for the object that should satisfy this relation. This region of interest allows to focus attention towards the only region satisfying the relation (to some degree). Since usually several relations are represented in the model for describing one structure, fusion of these representations should be performed. The fuzzy set framework offers a large set of fusion operators, varying from conjunctive to disjunctive ones, including adaptive operators [17]. The fusion of all regions of interest leads to a fuzzy region representing the combination of all relationships concerning one structure. Then segmentation of the structure can be based on image information (typically grey levels) focused in the obtained fuzzy region.

A recognition procedure based on this type of representation has been developed for the recognition of internal brain structures in MRI [18, 19]. The model has an iconic part (digital atlas) and a symbolic part (linguistic descriptions of relationships between anatomical structures). The procedure consists in recognizing first simple structures (typically brain and lateral ventricles), and then progressively more and more difficult structures, based on relationships between these structures and previously recognized structures. Each relationship describing the structure to be recognized is translated into a spatial fuzzy set representing the area satisfying this relation, to some degrees. The fuzzy sets representing all relationships involved in the recognition process are fused using a numerical fusion operator. In the obtained fuzzy region of interest, a segmentation procedure is performed, and the quality of the results is guaranteed by the very restricted (focused) area in which the structure of interest is searched. This approach typically belongs to the field of spatial reasoning under imprecision in images.

For instance, the recognition of a caudate nucleus in a 3D MRI image uses the results of recognition of brain and lateral ventricles and the following pieces of knowledge, illustrated in Figure 2:

– rough shape and localization are provided by the representation of the caudate nucleus in the atlas, and its fuzzy dilation to account for variability and for inexact matching between the model and the image,

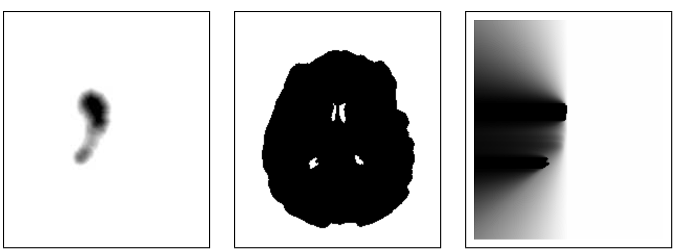

Fig. 2. Knowledge representation in the image space (only one slice of the 3D volume is shown), illustrating knowledge about one caudate nucleus: shape information (left), set relationships (middle), and relative directional relationship (right). Membership values vary from 0 (white) to 1 (black).

– the caudate nucleus belongs to the brain (black) but is outside from both lateral ventricles (white components inside the brain),
– the caudate nucleus is lateral to the lateral ventricle.

These pieces of knowledge are then combined (also with information extracted from the image itself), which leads to a successful recognition of the caudate nucleus.

Figure 3 illustrates the spatial representation of some knowledge about distances, used for several structures.

Figure 4 shows 3D views of some cerebral objects as defined in the atlas and as recognized in an MR image with our method. They are correctly recognized although the size, the location and the morphology of these objects in the image

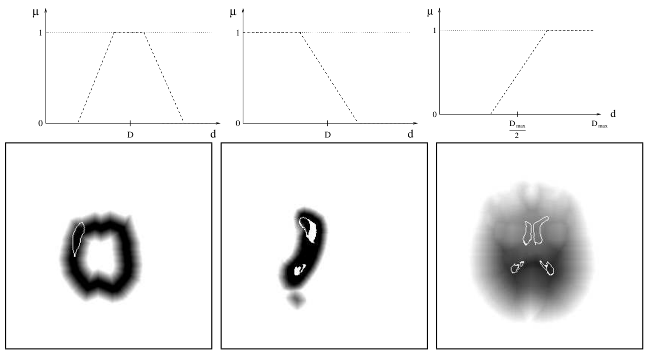

Fig. 3. Representation of knowledge about distances. Top: membership functions μ_n. Bottom: spatial fuzzy sets. The following examples are illustrated: the putamen has an approximately constant distance to the brain surface (left), the caudate nucleus is at a distance about less than D from the lateral ventricles (in white) (middle), lateral ventricles are inside the brain and at a distance larger than about D from the brain surface (right). The contours of the objects we are looking at are shown in white.

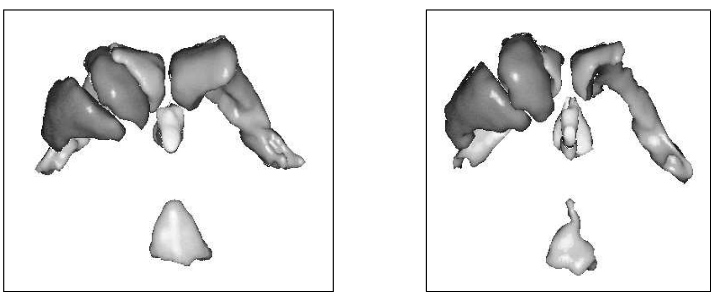

Fig. 4. Recognition results. The left view represents six objects from the model atlas: lateral ventricles (medium grey), third and fourth ventricles (light grey), caudate nucleus and putamen (dark grey). The right view represents the equivalent objects recognized from a MRI acquisition. (From [18].)

significantly differ from their definitions in the atlas. Note in particular the good recognition of third and fourth ventricles, that are very difficult to segment directly from the image. Here the help of relationships to other structures is very important.

The segmentation can be further improved once recognition is achieved by integrating the fuzzy regions representing the spatial relations as new energy terms in deformable models [20].

This approach has been used in other domains, for instance in mobile robotics to reason about the spatial position of the robot and the structure of its environment [21].

4 Conclusion

As illustrated in this paper, the semi-qualitative fuzzy set framework shows interesting features both for knowledge representation (of spatial relations, of imprecision existing both in the objects and in the relations), and for reasoning and recognition. We have also shown the usefulness of fuzzy mathematical morphology in this context. This work opens new perspectives for spatial reasoning under imprecision in image interpretation.

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