A Fuzzy Mathematical Morphology Approach to Multiseeded Image Segmentation

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Abstract. We propose an innovative segmentation algorithm based on mathematical morphology operators. This definition is based on a morphological and fuzzy pattern-matching approach, and consists in comparing an object to a fuzzy landscape representing the degree of satisfaction of an affinity relationship. It has good formal properties, it is flexible, it fits the intuition, and it can be used for structural pattern recognition under imprecision. Moreover, it also applies in 3D and for fuzzy objects issued from images.

1 Introduction

The spatial arrangement of objects in images provides important information for recognition and interpretation tasks, in particular when the objects are embedded in a complex environment like in medical images. Relationships between objects, in particular for image segmentation purposes, can be described in terms of affinity between them, and it is the aim of this paper to address the problem of defining such relationships. From our every day experience, it is clear that any all-or-nothing definition leads to unsatisfactory results in several situations, even of moderate complexity. Fuzzy approaches are all the most interesting when imprecision in images has to be taken into account. Indeed, the representation of image regions as spatial fuzzy sets is useful to take into account the imprecision inherent to images. Several different image segmentation methods have been proposed in literature in the past [6]. In this paper we propose a new approach based on mathematical morphology [3].

In the present work, we show how the shape of objects in the image can be utilized to define a new segmentation algorithm. This method has been inspired on a work about fuzzy relative position between objects according to morphological operators [1]. Indeed, this algorithm is based on the concept of affinity. Affinity is a fuzzy relation defined between pixels of the image and its goal is to capture the grade of their "hanging togetherness".

The paper is organized as follows: Section 2 is dedicated to the basic concepts, specifically inherently to the fuzzy affinity; Section 3 presents how affinity can

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be used with some morphological operators for the formulation of a segmentation algorithm; Section 4 describes the proposed algorithm, finally Section 5 the performances evaluation.

2 Fuzzy Affinity

Let us represent the image domain by S and let $\mathcal{S} = (S, f)$ be a fuzzy scene, where f is the pixel intensity function. We define a fuzzy relation κ in S, with its membership function μ_κ; we would want κ to be such that $\mu_\kappa(c, d)$ is a function of $V(c, d)$, that is the neighbours of both c and d and of $f(c)$ and $f(d)$, that is the intensity features. We can use the following functional form for μ_κ,

$$\mu_\kappa(c, d) = g(\mu_\psi(c, d), \mu_\phi(c, d)). \tag{1}$$

ψ and ϕ respectively represent the *homogeneity-based* and the *object-feature-based* components of affinity and μ_ψ and μ_ϕ the respective membership functions. The strenght of relation ψ indicates the degree of local hanging togetherness of spels because of their intensity similarities. The strenght of relation ϕ indicates the degree of local hanging togetherness of pixels because of the similarity of their features values to some(specified) object feature. The function g can be considered as a fusion operator; we have chosen an average type fusion operator, which achieves a compromise between both pieces of information:

$$g = \sqrt{\mu_\phi \mu_\psi}. \tag{2}$$

For the homogeneity-based affinity, we assume the following expression:

$$\mu_\psi(c, d) = W_\psi(|f(c) - f(d)|). \tag{3}$$

In our work, μ_ψ is assumed as a Gaussian function with zero mean:

$$W_\psi(x) = e^{\frac{x^2}{2k_\psi^2}}, \text{with} x = |f(c) - f(d)|.$$

The treatment of μ_ϕ is somewhat different from that of μ_ψ. We consider the object feature as well as background feature to formulate μ_ϕ. We use an object membership function W_o, as well as background membership function W_b, to capture the idea of membership of any pixel to the respective regions and then combine them to obtain μ_ϕ.

For our purpose, we choose them as Gaussian functions. Namely, we set $W_o(x) = e^{\frac{(x-m_o)^2}{2k_o^2}}$ and $W_b(x) = e^{\frac{(x-m_b)^2}{2k_b^2}}$.

Finally, we consider any points c and d to have a high *object-feature-based* affinity only if both c and d have high object membership (i.e., the value of W_o) and both have low background membership (i.e., the value of W_b). The functional form chosen to reflect this strategy is as follows:

$$\mu_\phi(c, d) = \begin{cases} 1\,, & \text{if } c = d \\ \frac{\mathcal{W}_o(c,d)}{\mathcal{W}_o(c,d) + \mathcal{W}_b(c,d)}\,, & \text{otherwise} \end{cases} \tag{4}$$

where

$$\mathcal{W}_o(c,d) = \min[W_o(f(c)), W_o(f(d))], \tag{5}$$

and

$$\mathcal{W}_b(c,d) = \max[W_b(f(c)), W_b(f(d))]. \tag{6}$$

3 Segmentation Algorithm

Let us consider an image, whose domain is indicated by S, and a set of reference objects in it O_i, for $i = 1, 2, ..., n$ where n is the number of desired objects. These objects could be considered as binary or fuzzy. In the last case a specific fuzzy relation o_i [1] is defined on each of them, as indicated in Section 3.1; the related membership function $\mu_{o_i}(c)$ indicates, for some pixel $c \in S$, the degree of satisfaction of the specific fuzzy relation o_i.

In order to establish the relationships between the binary or fuzzy objects O_i and S for segmentation purpose, we choose the following approach:

1. We first define a fuzzy "landscape" μ_{f_i}, around the reference objects O_i, as fuzzy sets such that the membership values of each point, that is μ_{f_i}, corresponds to the degree of satisfaction of the fuzzy relation κ defined before.
2. We then compare S to the fuzzy landscapes μ_{f_i} in order to evaluate how well a specific object O_i matches S. This is done using a fuzzy pattern-matching approach.

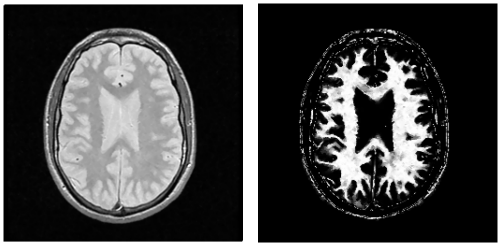
3.1 Definition of the Fuzzy Landscape

The goal of the definition of the fuzzy landscape is to point out the relations of affinities between pixels of the image.

The definition of the fuzzy landscape can be adapted to binary and fuzzy objects; for our purpose, we have only considered fuzzy objects. The fuzzy objects have been defined by means of a specific fuzzy relation o_i, "**degree of membership to a specific manually selected object** O_i". The membership function μ_{o_i}, for each of the selected objects O_i, has been defined by means of a Gaussian funtion whose mean and variance are related to the mean and variance of the pixels of the selected regions of the image. So, given an object O_i[2], each point $c \in S$ is characterized by its "**degree of affinity with object** O_i". The affinity relation is the one defined before, indicated by κ. As pointed in Section 4, we compute several different affinity relations κ_i, one for each of the object O_i we want to segment.

[1] O_i and o_i are two different notations: the first one indicates the reference objects while the last one the fuzzy relations defined on them.

[2] In our application, an operator selects, on a display of a slice of the scene, for each object O_i, a region of the object and a region of the background using a mouse-controlled brush.



(a) MRI (b) fuzzy landscape of (a)

Fig. 1. Illustration of the concept of the fuzzy landscape of a MRI image: (a) original image, (b) fuzzy landscape of (a)

In establishing the fuzzy landscape μ_{f_i} of a specific object O_i, in fuzzy case, we choose a method that combines directly μ_{o_i}, describing the membership to some O_i, with the strenght of affinity μ_κ [1]. In fuzzy terms the following holds:

$$\mu_{f_i}(c) = \max_{d \in Supp(O_i)} t[\mu_{o_i}(c), \mu_{\kappa_i}(c,d)], \tag{7}$$

where t is a t-norm. This definition can be adapted to the case of multiple objects, in that case $\mu_\kappa(c,d)$ is simply replaced by $\mu_\kappa^{(i)}(c,d)$, for $i = 1, 2, ...n$.

In Fig.1(a) and Fig.1(b) is illustrated the concept of fuzzy landscape. It is an interesting rapresentation of the fuzzy landscape for MRI image. Fig.1(a) shows an original a MRI image, Fig.1(b) represents the fuzzy landscape according to the fuzzy relation "**degree of affinity to the gray matter**". It's clear that points of Fig.1(b) that are embedded in the gray matter have high membership values (white pixels of the images).

3.2 Fuzzy Pattern Matching

The fuzzy landscape defined before allows us to define objects in the image based on some specific characteristic of affinity. The process of objects extraction, for segmentation purpose, is determined by means of the evaluation of the degree of matching between S and the fuzzy landscape of the objects O_i obtained from the previous step.

Let us denote by μ_S the membership function of S, which is a function of S in $[0,1]$, where S is the image domain. An appropriate tool for defining the degree of matching of S with respect to each fuzzy landscape μ_{f_i} is the fuzzy pattern-matching approach [2]. Following this approach, the evaluation of the matching between two possibility distributions consists of two numbers, a necessity degree Π and a possibility degree N. Π and N are computed for each object O_i, $i = 1, \ldots, n$, according to the following expressions:

$$\Pi_i(x) = \sup_{y \in S} t[\mu_{f_i}(y-x), \mu_S(y)] \quad \forall x \in S \tag{8}$$

$$N_i(x) = \inf_{y \in S} T[\mu_{f_i}(y-x), 1-\mu_S(y)] \quad \forall x \in S \tag{9}$$

where t is a t-norm (fuzzy intersection) and T a t-conorm (fuzzy union) [3]. In the crisp case, these equations reduce to:

$$\Pi_i(x) = \sup_{y \in S} \mu_{f_i}(y) \quad \forall x \in S \tag{10}$$

$$N_i(x) = \inf_{y \in S} \mu_{f_i}(y) \quad \forall x \in S \tag{11}$$

The possibility and necessity can be interpreted in terms of fuzzy matheatical morphology, since the possibility is equal to the dilation of μ_S by μ_{f_i}, while the necessity is equal to the erosion [3].

4 The Proposed Algorithm

In this section we review the main steps of the proposed segmentation algorithm.

As first step, the user must select on the image, for each of the objects O_i he desires to segment, an object and a background region. These regions are used as statistical samples of training for the computation of the parameters of the affinity, that is m_o and m_b for the mean and κ_o, κ_b and κ_ψ for the standard deviation.

The second step is represented by the computation of the fuzzy landscapes μ_{f_i} for each of the the desired objects O_i. We propose here a fast algorithm for computing them.

The algorithm consists in performing two passes on the image, one in the conventional sense, and one in the opposite sense. For each point c, we store the point $Q = O(c)$ from which the maximum affinity is obtained. For a point c, we don't consider all points in O_i as for exhaustive method, but only those of neighbourhood of c. Specifically, we compute the fuzzy landscape as:

$$\mu_{f_i}(c) = \max_{d \in V(c)} t[\mu_{f_i}(O(d)), \mu_S(O(d))],$$

where $V(c)$ denotes the neighbourhood of c. Let d_c be the point d for which the maximum affinity value is obtained

$$d_c = arg\max_{d \in V(c)} t[\mu_{f_i}(O(d)), \mu_S(O(d))]$$

Then, we set: $O(c) = O(d_c)$.

As final result of the second step, each point c of the image is characterized by its membership, its degree of affinity with each of the manually selected objects O_i, that is $\mu_{f_i}(c)$, $\forall c \in S$ and $\forall i = 1, 2, ...n$. Then, as pointed before in Section 3, the fuzzy landscapes of the objects are matched with S. This is realized by means of the computation of the values Π_i and N_i. As in the computation of the fuzzy landscape, this is realized $\forall c \in S$ and $\forall i = 1, 2, ...n$. The computation of $\Pi_i(c)$ and $N_i(c)$, as regards fuzzy case, is realized as indicated in (10) and (11). The final decision is to assign a point $c \in S$ to an object for which it has the maximum degree of matching.

The computational time of the algorithm is $O(cn_c n)$, where c is the number of pixels, $n_c = |V(c)|$, that is the number of neighbours of c, and n the number of objects being extracted. This algorithm is quite fast with respect to the one of the Fuzzy Connectednees in [7]. Even if the computational time is quite similar ($O(c(n+1))$) the differences are in operational costs, that is in terms of ms.

Below we propose the final image segmentation algorithm:

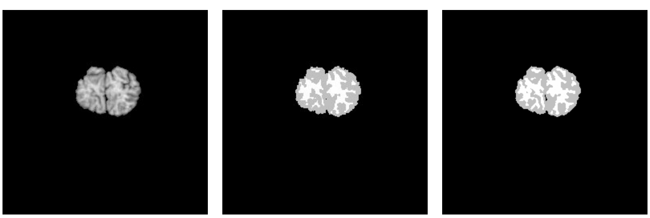
1. Estimate parameters for affinity.
2. **for** each object i **do**
3.   **for** each pixels c **do**
4.   compute $\mu_{f_i}(c)$ according to (7);
5. **for** each object i **do**
6.   **for** each pixels c **do**
7.   compute $g_i(c) = \frac{\Pi_i(c)+N_i(c)}{2}$ according to (10) and (11);
8. **for** each pixels c **find**
9. $p_i = arg\max g_i(c)$;
10. Output p_i.

5 Experimental Results

In order to evaluate the experimental results of the algorithm, we have tested it segmenting MRI images. Based on [4], we have used 100 MRI images of the IBSR (http://www.cma.mgh.harvard.edu/ibsr/). The results obtained by this algorithm have been compared to the results of the Fuzzy Connectedness segmentation algorithm [7]. We have used the SSIM-INDEX algorithm in order to evaluate the degree of similarity between the original signal (groundtruth) and the distorted signal (segmented image), [9].

In Fig.2(b) is shown an example of segmentation. The groundtruth is shown in Fig.2(c), while the original images in Fig.2(a).

In Table 1(a) are shown the results obtained by the proposed algorithm on the 100 MRI images of the IBSR. Each row of the tables reports the results of 5 images. We obtain a 90% of similarity that is comparable with results achieved on the same data set by similar algorithms based on fuzzy connectivity [5].



(a) (b) (c)

Fig. 2. Segmentation obtained by the proposed algorithm: (a) original image , (b) segmented image, (c) groundtruth

Table 1. Performances evaluation

Images - SSIM-INDEX					
1-5	0.9691	0.9621	0.9616	0.9474	0.9227
6-10	0.9130	0.8992	0.9079	0.8905	0.8882
11-15	0.8806	0.8790	0.8718	0.8675	0.8522
16-20	0.8564	0.8493	0.8524	0.8541	0.8698
21-25	0.8781	0.8610	0.8676	0.8598	0.8465
26-30	0.8572	0.8481	0.8515	0.8595	0.8605
31-35	0.8606	0.8673	0.8768	0.8709	0.8787
36-40	0.8743	0.8908	0.8979	0.9082	0.9147
41-45	0.9263	0.9238	0.9318	0.9330	0.9379
46-50	0.9335	0.9425	0.9453	0.9482	0.9593
Mean 0.8942					

(a) SSIM-INDEX of the proposed algorithm for the first 50 images.

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