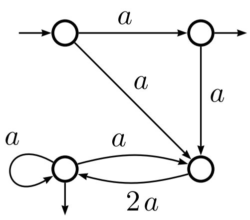
Lecture III — Exercises

Unless stated otherwise, the alphabet A is $A = \{a, b\}$.

1. Compute the reduced representation of the following $\mathbb{N}$-automaton.



2. Let $\mathcal{A}_1$ be the $\mathbb{Q}$-automaton on $\{a\}^*$ shown at Figure 1 (the unique letter a of the alphabet is not shown on the transitions of the figure). Compute a reduced automaton, equivalent to $\mathcal{A}_1$.

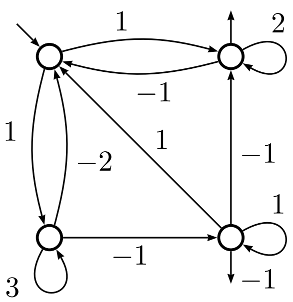
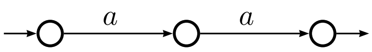


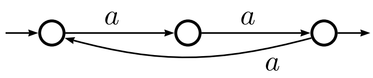
Figure 1: The $\mathbb{Q}$-automaton $\mathcal{A}_1$

3. Consider the minimal (Boolean) automaton of $\{a^n \mid n \equiv 0, 1, 2, 4 \pmod 7\}$ as an automaton with multiplicity in $\mathbb{Z}/2\mathbb{Z}$ and reduce it. Comment.

4. Let $\mathbb{F}$ be a field. Show that two $\mathbb{F}$-recognisable series over A^* are equal if and only if they coincide on all the words of length less than the sum of the dimensions of the representations which realise them.

Show the bound is sharp. [*Hint: consider the following two automata.*]





5. **Discriminating length.** We call the *discriminating length* between two non-equivalent (Boolean) automata $\mathcal{A}$ and $\mathcal{B}$ the length of a shortest word which is accepted by one and not the other. We write $L_{\mathsf{d}}(n, m)$ (resp. $L_{\mathsf{nd}}(n, m)$) for the maximum of the discriminating lengths when $\mathcal{A}$ and $\mathcal{B}$ have respectively n and m states and are deterministic (resp. and are non-deterministic).

(a) With methods relevant to Boolean automata, show that $L_{\mathsf{d}}(n, m) \leqslant n\,m$.

(b) Compute $L_{\mathsf{d}}(n, m)$.

(c) Give an upper bound for $L_{\mathsf{nd}}(n, m)$.